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An approach to constrained polynomial optimization via nonnegative circuit polynomials and geometric programming

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ABSTRACT

In this article we combine two developments in polynomial optimization. On the one hand, we consider nonnegativity certificates based on sums of nonnegative circuit polynomials, which were recently introduced by the second and the third author. On the other hand, we investigate geometric programming methods for constrained polynomial optimization problems, which were recently developed by Ghasemi and Marshall. We show that the combination of both results yields a new method to solve certain classes of constrained polynomial optimization problems. We test the new method experimentally and compare it to semidefinite programming in various examples.

1. Introduction

Solving polynomial optimization problems is a key challenge in countless applications like dynamical systems, robotics, control theory, computer vision, signal processing, and economics; e.g.

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Blekherman et al., 2013; Lasserre, 2010. It is well-known that polynomial optimization problems are NP-hard in general; see e.g., (Laurent, 2009). Starting with the seminal work of Lasserre (2000/01), relaxation methods were developed which are significantly faster and provide lower bounds. These methods were studied intensively by means of aspects like exactness and quality of the relaxations (de Klerk and Laurent, 2010; Nie, 2013a, 2013b, 2014), the speed of the computations (Lasserre, 2010; Parrilo and Sturmfels, 2003), and geometrical aspects of the underlying structures (Blekherman, 2006, 2012). A great majority of these results are based on the original approach by Lasserre, called *Lasserre relaxation*, which relies on *semidefinite programming (SDP)* methods and *sums of squares (SOS)* certificates to provide lower bounds for polynomial optimization problems. SDPs can be solved in polynomial time in problem size (up to an ε-error); e.g. Blekherman et al. (2013, p. 41) and references therein. However, the size of such programs grows exponentially with the number of variables n or the degree d of the polynomials, as its size is given by the number of monomials of n-variate monomials of degree at most d.

Recently, Ghasemi and Marshall suggested a promising alternative approach both for constrained and unconstrained optimization problems based on *geometric programming (GP)* (Ghasemi and Marshall, 2012, 2013). GPs can also be solved in polynomial time (up to an ε-error) (Nesterov and Nemirovskii, 1994); see also Boyd et al. (2007, page 118), but, by experimental results, e.g. Boyd et al., 2007; Ghasemi and Marshall, 2012, 2013; Ghasemi et al., 2014, in practice the corresponding geometric programs can be solved *significantly* faster than their counterparts in semidefinite programming. The lower bounds obtained by Ghasemi and Marshall are, however, by construction worse than lower bounds obtained via semidefinite programming, and they can only be applied in very special cases.

Independent of Ghasemi and Marshall, the second and the third author recently developed a new certificate for nonnegativity of real polynomials called *sums of nonnegative circuit polynomials (SONC)* (Iliman and de Wolff, 2016a). SONC certificates are independent of SOS certificates. In Iliman and de Wolff (2016b) the second and third author showed that the GP based approach for unconstrained optimization by Ghasemi and Marshall can be generalized crucially via SONC certificates. In consequence, the presented geometric programs are linked to sums of nonnegative circuit polynomials similarly as semidefinite programming relaxations are linked to sums of squares. Particularly, there exist various classes of polynomials for which the GP/SONC based approach is not only *faster* but, it also yields *better* bounds than the SDP/SOS approach. The reason is that all certificates used by Ghasemi and Marshall are always SOS, while SONCs are not SOS in general; see Iliman and de Wolff (2016a, Proposition 7.2).

The first contribution of this article is an extension of the results in Iliman and de Wolff (2016b) to constrained polynomial optimization problems. We focus on the class of *ST-polynomials*, that are polynomials which have a Newton polytope that is a simplex and which are satisfying some further conditions; see Section 2.1. The starting point is a general optimization problem from Iliman and de Wolff (2016b, Section 5), see (2.6), which provides a lower bound for the constrained problem but which is not a geometric program. Using results from Ghasemi and Marshall (2013), we relax the program (2.6) into a geometric optimization problem; see program (3.2) and Theorem 3.1. Additionally, we show in Theorem 3.4 that (2.6) can always at least be transformed into a signomial program; see Section 2.2 for background information. Furthermore, we prove that the new, relaxed geometric program (3.2) provides bounds as good as the initial program (2.6) for certain special cases, see Theorem 3.5.

In Section 4, we provide examples testing our new program (3.2) in practice and comparing it with semidefinite programming. Moreover, we demonstrate that increasing the degree of a given problem has almost *no* effect on the runtime of our program (3.2). This is in sharp contrast to SDPs, where one can nullify increased runtimes induced by high degrees only by additional pre-processing methods, e.g. by exploiting sparsity.

Furthermore, a bound obtained by Ghasemi and Marshall in Ghasemi and Marshall (2013) can never be better than the bound given by the d-th Lasserre relaxation for some specific d determined by the degrees of the involved polynomials. In Section 4 we provide examples showing that our program (3.2) can provide bounds which are better than the particularly d-th Lasserre relaxations.