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A possible unification of Newton's and Coulomb's forces

J.C. Caillon

Centre d'Etudes Nucléaires de Bordeaux-Gradignan, CNRS-IN2P3, Université de Bordeaux, Le Haut-Vigneau, 33170 Gradignan Cedex, France

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ABSTRACT

We have considered electric charge as the fourth component of the particle momentum in five-dimensional space–time. The fifth dimension has been compactified on a circle with an extremely small radius determined from the fundamental physics constants. First, we have given equations in the framework of five-dimensional special relativity and determined the corresponding reduction to four-dimensional space–time. Then, in order to obtain an appropriate charge-to-mass ratio and to avoid the Fourier modes problem, we have considered the propagation of an off-mass shell particle in the five-dimensional space–time which can be interpreted as the motion of an on-mass shell particle in the four-dimensional world we experience. As an example, we have discussed the five-dimensional kinematic equations associated with the electron-positron annihilation process into two photons. Finally, the consequences on the gravitational interaction between two elementary charged particles has been studied. As a main result, we have obtained a unification of Newton's gravitational and Coulomb's electrostatic forces.

1. Introduction

The gravitational interaction is described as pure geometry by the general relativity theory while the electromagnetic interaction is a renormalizable quantum field theory governed by local $U(1)$ gauge invariance. These two theories are extremely different by nature and it is a complicated task to marry them in a unified theory. On the other hand, classically Newton's gravitational and Coulomb's electrostatic forces seem so similar with $1/r^2$ laws (so infinite range interactions) and thus with identical properties like for example Gauss's theorem. However, these similarities hide deep differences, in particular, the strength of the two interactions (Newton's gravitational force between two charged particles is many orders of magnitude smaller than the electrostatic force) as well as for the nature of their sources: mass and electric charge.

While the Higgs mechanism contributes to a better understanding of the origin of mass of elementary particles, the nature of electric charge is entirely unknown. Nobody knows why its elementary value is the same for a point-like lepton and an infinitely more complicated composite proton. So, electric charge is just seen as an intrinsic property of a particle with two fundamental aspects: a conservation law and quantization. This is a conserved quantity because of the $U(1)$ phase invariance (no absolute phase). Other conservation laws also exist as those associated with dynamic quantities. It is well known that energy and momentum conservation arise from demanding invariance under respectively time and space translations (no absolute position in space–time). Energy and momentum are thus associated with the four space–time dimensions while this is not actually the case for electric charge. However, in the *CPT* theorem, spatial (related to P) and temporal (related to T) dimensions as well as electric charge (related to C) seems be on an equal footing. Since the nature of the latter is unknown, one can wonder if it couldn't also be related to a dimension and more particularly to an extra dimension.

This idea first appears in 1914 before general relativity with the work of Nordstrom [1] giving a 5D unified field theory of Newtonian gravity and electromagnetism. In 1921, Kaluza [2] considered the extension of general relativity to five dimensions. With the assumption that physics does not depend on the extra dimension (cylinder condition), he obtained that general relativity, when interpreted as a five-dimensional theory in vacuum, contained four-dimensional general relativity in the presence of an electromagnetic field together with Maxwell's laws of electromagnetism (plus a Klein-Gordon equation for a massless scalar field). A few years latter, to explain why physics depends on the first four coordinates but not on the extra one, Klein [3] in 1926 suggested that the fifth dimension is compactified on a circle with a very small radius roughly equal to the Planck length. It seemed early on that the expansion of the electromagnetic field into Fourier modes could in principle explain the quantization of electric charge. Moreover, with the extension to matter fields, electric charge was identified with the motion of the particle in the fifth dimension (see for example [4–6]). But these aspects have been little appreciated be-

E-mail address: caillon@cenbg.in2p3.fr.

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cause the charge-to-mass ratio did not match that of any known particles [5,7]. So nowadays, in compactified extra dimension theories (see for example [8,9]), Standard Model (SM) fermions are generally identified with the ground state Fourier modes [5], i.e. no motion in the fifth dimension.

On the other hand, there have been concurrent models that do not make use of Klein's compactification paradigm (or any assumptions about the size of extra dimensions). For example, in Space–Time–Matter theory [10,11] one postulates that the 4D curvature arises because our universe is embedded in some higher-dimensional vacuum manifold and geometric features from this embedding appear to be real matter (which has then a geometrical origin). In this theory, the fifth dimension is not temporal or spatial but mass-like, i.e. position in the fifth dimension is related to the mass of the particle. In membrane theory (see for example the ADD model [12] or the RS model [13]), the membrane is a hypersurface which concentrates the interactions of particles except gravity which can freely propagate outside in the bulk. However, even if originally only gravity was assumed to propagate in the extra dimensions, it was soon clear [14,15] that the Standard Model gauge fields and fermions could also propagate in the five-dimensional space–time.

In this work, we would like to consider the possibility that elementary charged particles propagate in the five-dimensional (5D) space–time with a compactified fifth dimension. In addition, we are interested in identifying electric charge as the fourth component of the particle momentum, i.e. the fifth component of the five-vector energy-momentum (section 2). This hypothesis could allow us to take into account mass and charge through a same physical quantity (the five-vector energy-momentum) and perhaps show a possible link between Newton's gravitational and Coulomb's electrostatic forces (through a common formalism for their sources). Since the fifth dimension is not observed, it has been considered as compactified on a circle with an extremely small radius whose the value has been determined from the fundamental physics constants (section 3). We have first considered the propagation of a particle in the framework of 5D special relativity (subsection 4.1). In this framework, the phenomena of time dilation finds a simple explanation when associated with our initial assumption on electric charge. The motion of an elementary particle in 5D space–time appears to us as a particle moving in four-dimensional (4D) space–time with a modified mass taking into account the unobserved fourth component of the particle momentum. As in the Kaluza–Klein theory, we have found an enormous mass value for an elementary particle. In order to avoid this well known charge-to-mass ratio problem, we have then considered the propagation of an off-mass shell particle in 5D space–time (subsection 4.2). It can be interpreted, in our four-dimensional world, as the motion of an on-mass shell particle with the appropriate charge-to-mass ratio. Elementary charged fermions are here identified with the first mode, $n = \pm 1$, and higher values of $|n|$ are found associated with enormous mass values not observed in our low-energy world. In order to show how processes can be interpreted in this framework, as an example, we have discussed the kinematic equations for the electron-positron annihilation process into two photons. Finally, we have determined the gravitational interaction between two particles propagating in the 5D space–time (section 5). As a main result, we have obtained a unification of Newton's gravitational and Coulomb's electrostatic forces.

2. Electric charge as the fourth component of the particle momentum

As mentioned in the introduction, we start with the assumption that electric charge q is a constant quantity associated with a small extra compact dimension, noted w. In the 5D space–time, the phase term of the wave function for a particle with energy E and three-momentum $\boldsymbol{p} = (p_x, p_y, p_z)$ can be written as:

$$e^{-\frac{i}{\hbar}(p^\mu x_\mu)}, \tag{1}$$

where $x^\mu = (ct, \boldsymbol{r}, w)$ and $p^\mu = \left(\frac{E}{c}, \boldsymbol{p}, p_w\right)$ are here five-vectors respectively for position and energy-momentum (with $\boldsymbol{r} = (x, y, z)$). We suppose that:

$$p_w = \frac{q}{a}, \tag{2}$$

where the dimensional coefficient a allows the fifth component of p^μ to have momentum units as c does for energy E. Note that we have two ways of defining electric charge according to whether we consider the covariant or contravariant components of p_w. Such ambiguity is also found precisely in the definition of electric charge since by convention we associate the negative elementary charge for electron. To determine the coefficient a, we have taken into account the Planck units which provides natural units obtained from the fundamental physic constants. First of all, note that the speed of light (arising in the first component of p^μ) is also obtained from the Planck units since:

$$c = \frac{l_p}{t_p}, \tag{3}$$

where $l_p = \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-35}$ m and $t_p = \sqrt{\frac{\hbar G}{c^5}} = 5.39 \times 10^{-44}$ s are respectively the Planck length and the Planck time. In the same way, using Eq. 1, we obtain a by:

$$a = \frac{l_p q_p}{E_p t_p} = \frac{\sqrt{4\pi\varepsilon_0 G}}{c} = 2.88 \times 10^{-19}\,\mathrm{A\,kg^{-1}\,s^2\,m^{-1}}, \tag{4}$$

where $E_p = \sqrt{\frac{c^5\hbar}{G}} = 1.96 \times 10^9$ J and $q_p = \sqrt{4\pi\varepsilon_0\hbar c} = 1.88 \times 10^{-18}$ C are respectively the Planck energy and the Planck charge.

As already mentioned in Kaluza–Klein theory [5,6], the two possible signs of electric charge can be understood as the consequence of the two possible directions of motion around the closed fifth dimension. In addition, the conservation law of electric charge in all known reactions comes naturally from conservation of the momentum. Thus, we can understand why there is no annihilation between two electrons while this is possible between an electron and a positron. Indeed, the former is not possible because of the non-conservation of the fifth component of the five-vector energy-momentum. We also can re-interpret the *CPT* theorem, where time reversal T transforms $t \to -t$, parity P transforms $\boldsymbol{r} \to -\boldsymbol{r}$ and charge conjugation transforms $q \to -q$ (as well as other charges). Indeed, in the 5D version of CPT theorem we have only *PT* but with P which transforms both $\boldsymbol{r} \to -\boldsymbol{r}$ and $w \to -w$. However, reversing the direction of the fifth dimension is equivalent to changing the sign of p_w. This allows us to understand why spatial and temporal dimensions as well as electric charge are together in the *CPT* theorem.

3. Compactified fifth dimension

Since the extra dimension is compactified on a circle with a radius R, the wave function ϕ must then be periodic in the coordinate w along this direction [3]:

$$\phi\,(ct, \boldsymbol{r}, w) = \phi\,(ct, \boldsymbol{r}, w + 2\pi R)\,. \tag{5}$$

This means that, without loss of generality, the wave function can be expanded in a Fourier series in w:

$$\phi\,(ct, \boldsymbol{r}, w) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi^{(n)}\,(ct, \boldsymbol{r})\, e^{\frac{inw}{\hbar R}}\,, \tag{6}$$

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