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Nonlocality and local causality in the Schrödinger equation with time-dependent boundary conditions

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ABSTRACT

We investigate the nonlocal dynamics of a single particle placed in an infinite well with moving walls. It is shown that in this situation, the Schrödinger equation (SE) violates local causality by causing instantaneous changes in the probability current everywhere inside the well. This violation is formalized by designing a gedanken faster-than-light communication device which uses an ensemble of long narrow cavities and weak measurements to resolve the weak value of the momentum far away from the movable wall. Our system is free from the usual features causing nonphysical violations of local causality when using the (nonrelativistic) SE, such as instantaneous changes in potentials or states involving arbitrarily high energies or velocities. We explore in detail several possible artifacts that could account for the failure of the SE to respect local causality for systems involving time-dependent boundary conditions.

1. Introduction

Nonlocality is the hallmark of quantum mechanics. It is generally taken for granted that nonlocality requires two or more particles, along the lines of the early paper by Einstein Podolsky and Rosen [1], subsequently put into a firm footing by Bell [2]. Although it has been suggested that a single particle could in some instances exhibit nonlocality, such results have been disputed. This is particularly the case of the two main candidates for single particle nonlocality, the Aharonov-Bohm effect [3] and the entanglement between spatial modes of a single photon (see [4] and Refs. therein for previous works), discussed respectively in Refs. [5,6] and [7,10].

The present work introduces a new "candidate" for single particle nonlocality. It is based on the fact that the Schrödinger equation solved on a domain with moving boundaries gives rise to apparent violations of local causality. It appears that time-dependent boundary conditions can potentially induce a nonlocal change in a region located far from the location of the moving boundary. Here we will examine the case of a particle in a box with infinitely high but moving walls. We will see that for quantum states extended all over the box, the moving walls generate instantaneously a current density almost everywhere in the box. We will indicate how this effect could be in principle tested, namely by making weak measurements of the particle momentum in the central region of the box before light has the time to propagate from the walls to that region. To this effect, a gedanken faster-than-light communication device will be presented.

Let us state right away that we are not advocating the position that it is possible to send a signal faster than the speed of light. Nevertheless, the present problem is interesting because the non-relativistic Schrödinger equation fails to prevent superluminal signaling in a situation where relativistic considerations do not seem to play a significant role. It is indeed well-known that the Schrödinger equation does not bound particle velocities, nor does it constrain instantaneous changes in potentials, but we will argue that in our system the nonlocal aspects do not rely on spurious violations of special relativity allowed by a employing a nonrelativistic framework.

Note that the effect reported in this work is not due to a non-dynamical phase term, such as a geometric phase (in which case we would have in the present context a non-adiabatic, non cyclic geometric phase [8,9]). There have been in the past claims that such non-dynamical phases in the same type of system that we will be investigating in this work could be envisaged as a specific form of "hidden" (i.e., non-signaling) non-locality [11–13]. We will see instead that the non-local aspect in our candidate system is not based on the existence of such phases.

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We will start by revisiting the treatment of systems with time-dependent boundary conditions of the form $\psi(x(t),t)=0$, where ψ is the wavefunction. Such systems are delicate to handle because from a formal point of view a different Hilbert space needs to be defined for each time t, so that a simple operation like taking the time derivative $\partial_t\psi$ is not straightforward. We will introduce the system we will deal with — a particle in an expanding infinite well — in the context of recent works [14–16] involving time dependent boundary conditions in Sec. 2.

Weak measurements were originally [17] introduced to measure an observable without significantly disturbing the system, allowing a subsequent standard (projective) measurement of a different observable. The outcome, known as a weak value, is not generally an eigenvalue (since the quantum state of the system is barely modified and no projection takes place) but still gives some information on the weakly measured observable, provided enough statistics are gathered by repeating the experience a certain number of times. In particular, it was shown [18] that the weak value of the momentum is directly related to the current density. We will recall these facts in Sec. 3 where we will present our main results concerning the instantaneous response of the current density to a change in the boundary conditions.

We will then proceed (Sec. 4) to analyze and discuss this novel type of nonlocality. The first issue we will address is no-signaling. No-signaling stands as the major constraint permitting the "peaceful coexistence" [19] of relativity and quantum mechanics. At first sight it would appear that no-signaling is respected here, since a single weak measurement does not convey any information, but the situation is more involved, and a protocol that would allow us to test in principle the possibility of signaling will be presented. Given that this nonlocal effect appears to conflict with the no-signaling principle, we will critically assess the origins of nonlocality, in search of possible artifacts. We will then discuss the present results in the framework of the Bohmian model, where nonlocality is a built-in feature claimed to hold for individual events but is washed out at the statistical level. A summary and our conclusions will be given in Sec. 5.

2. A particle in an infinite well with moving walls

The particle in an infinite well with moving walls was widely investigated in the context of quantum chaos (see e.g. [20–23]). Another line of studies concerning this system involves the conjecture of nonlocality induced by the moving wall on a localized state [12,24–29], that was recently disproved [16]. The Hamiltonian for a particle of mass m in an infinite well with the left wall fixed at $x=0$ and the right wall moving according to the function $L(t)$ is given by

$$H=\frac{P^2}{2m}+V \tag{1}$$

$$V(x)=\begin{cases}0 & \text{for } 0\le x\le L(t)\\ +\infty & \text{otherwise.}\end{cases} \tag{2}$$

The solutions of the Schrödinger equation $i\hbar\partial_t\psi(x,t)=H\psi(x,t)$ must obey the boundary conditions $\psi(0,t)=\psi(L(t),t)=0$. The instantaneous eigenstates of H,

$$\phi_n(x,t)=\sqrt{2/L(t)}\sin\left[n\pi x/L(t)\right] \tag{3}$$

verify $H|\phi_n\rangle=E_n(t)|\phi_n\rangle$ where $E_n(t)=n^2\hbar^2\pi^2/2mL^2(t)$ are the instantaneous eigenvalues, but, due to the time varying boundary conditions, the ϕ_n are *not* solutions of the Schrödinger equation. To solve the Schrödinger equation different approaches have been proposed, like introducing a covariant time derivative [30], implementing an ad-hoc change of variables [31], or relying on a time-dependent quantum canonical transformation [14,32]. Here we follow the latter option, as implemented in Ref. [16]. However, rather than going through the transformation to derive the solutions for the general case (this is done in [16]), we will choose from the beginning a specific function $L(t)$ for which analytic basis solutions of the Schrödinger equation are known. Indeed, for the linearly expanding case

$$L(t)=L_0+qt \tag{4}$$

it can be checked by inspection [31] that

$$\psi_n(x,t)=\sqrt{\frac{2}{L_0+qt}}\exp\left(-\frac{i\pi^2\hbar^2n^2t-iL_0m^2qx^2}{2\hbar mL_0\left(L_0+qt\right)}\right)\times\sin\left(\frac{n\pi x}{L_0+qt}\right) \tag{5}$$

verifies the Schrödinger equation and the boundary conditions $\psi(0,t)=\psi(L(t),t)=0$. Here, $q>0$ represents the velocity of the expanding wall.

The set of $\psi_n(x,t)$ (with n a positive integer) form a set of orthogonal basis functions useful to determine the time evolution of an initial arbitrary quantum state. The simplest initial state would be to pick a given $\psi_n(x,t=0)$; its evolution follows directly from Eq. (5). From a physical standpoint, it would be more realistic to start from the standard fixed wall eigenfunctions. A typical initial state would then be an eigenstate $\phi_n(x,t=0)$ [see Eq. (3)] or a linear combination thereof, say

$$\psi(x,t=0)=\sum_{n=1}^{\infty}c_n\phi_n(x,t=0) \tag{6}$$

whose evolution is given by

$$\psi(x,t)=\sum_{k,n}c_n\left\langle\psi_k(t=0)\right|\left.\phi_n(t=0)\right\rangle\psi_k(x,t). \tag{7}$$

We may want to include additional refinements, like allowing for a continuous transition from the fixed walls to the linear regime by setting

$$L(t)=L_0+qt(1-e^{-\gamma t}). \tag{8}$$

This requires numerical solutions. The numerical method that will be used here is very similar to the one exposed in Ref. [22]; it is based on looking for numerical solutions $\zeta(x,t)$ by using expansions over the instantaneous eigenstates of the form

$$\zeta(x,t)=\sum_{k=1}^{\infty}a_k(t)\phi_k(x,t). \tag{9}$$

The coefficients $a_k(t)$ are retrieved by solving a system (arising by plugging $\zeta(x,t)$ in the Schrödinger equation) of coupled differential equations.

3. Current density and momentum weak values

3.1. Current density evolution

We first briefly look at the standard current density

$$j=\frac{1}{2m}\left(\psi^*P\psi-\psi P\psi^*\right), \tag{10}$$

where P is the momentum operator for the states in an expanding infinite well. When the initial state is taken to be an eigenstate $\phi_n(x,0)$ of the fixed walls well, given by Eq. (3) with all the c_k

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