Research paper

A new multiscale phase field method to simulate failure in composites

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ABSTRACT

In this paper, a new multiscale phase field method (MsPFM) has been proposed to simulate crack propagation in composites. The MsPFM inherits the merits of anisotropic phase field method and multiscale finite element method. It is known that phase field simulation requires dense meshing to represent a sharp crack, thus the main aim of the MsPFM is to achieve mesh refinement in the vicinity of diffused crack/cracks. The proposed method is also used to study the interaction of a pre-existing crack with weak or strong interfaces (between matrix-fibre or between laminates) in terms of crack arrest, crack deflection, crack coalescence, and multiple cracks initiation in composites. Various numerical experiments are performed to demonstrate the effectiveness of proposed MsPFM to simulate the aforementioned failure characteristics of the composites. The crack growth trajectory obtained by the MsPFM for few test cases is validated through standard extended finite element method.

1. Introduction

The composite materials are the new generation materials, which possess varieties of extraordinary properties such as wear, corrosion, creep, thermal resistance, toughness, high strength-to-weight ratio, stiffness, high structural integrity to suit its applications in the mechanical, aerospace, robotics and biomedical fields. The global response of such materials is controlled by the mechanical properties and fatigue/fracture behavior of the interface between different constituents. In case of the weak fibre-matrix interface, the crack experiences crack arrest, repeated crack initiation, crack deflection, crack branching and crack coalescence. This complex fracture mechanism due to the presence of the weak interface increases the overall fracture toughness and load carrying capacity of the ceramic composite [1-3]. Hence, the numerical simulation of such complex fracture mechanism is not only a challenging task but also an interesting topic for the research community.

Over past decades, many numerical techniques have been developed for simulating localized failure (fracture) problems of the composite materials. Approaches like finite element method (FEM), boundary element method (BEM), extended finite element method (XFEM), element free Galerkin method (EFGM) and extended isogeometric analysis (XIGA) fall in the category of discrete approaches and models a sharp crack in the domain. Among these, FEM has been widely used to solve a variety of engineering problems including fracture. In spite of its numerous advantages and the huge success, it is found quite cumbersome for modeling crack growth due to conformal meshing and re-meshing requirement at each step of crack growth. To avoid the challenges faced by the FEM for crack growth modeling, [4] proposed EFGM, which was found quite suitable for crack growth simulations. Pant et al. [5] extended the EFGM to evaluate the stress intensity of interface cracks in bi-materials. On the similar line, Belytschko and his coworkers [6,7] developed the XFEM for solving the various crack propagation problems. In XFEM, the crack is modelled by enriching the displacement approximation through the partition of unity (PU). The enrichment for interface cracks was proposed by Sukumar et al. [8]. A major advantage of XFEM is the ability to model discontinuities without a use of conformal mesh [9]. Thus, this method is found quite useful in evaluating equivalent properties [10, 11], fatigue life prediction of the heterogeneous materials [9], and multiple cracks simulation [12-15]. To model crack growth in complex geometries, NURBS based enriched IGA known as extended isogeometric analysis (XIGA) have developed [16]. The XIGA was further implemented to perform the fatigue crack growth analysis in heterogeneous materials and bi-layered functionally graded materials (FGMs) [17]. Atroshchenko et al. [18] proposed generalized IGA for local mesh refinement.

Although many discrete approaches offered good accuracy in modeling crack growth however, they struggle in simulating crack initiation, handling multiple cracks and complex crack trajectories. The smeared/diffused crack approaches can overcome the issues faced by the discrete crack approaches. The nonlocal gradient damage approach and phase field approaches fall in the category of fully regularized...
diffused crack approach. The nonlocal gradient damage approaches [19,20] are based on the micromechanical model [21] in which a damage variable is incorporated in the constitutive model to represent the material softening and degradation. On the other hand, phase field method (PFM) is an energy-based approach, where a damage variable is incorporated in its variational formulation [22] to model brittle fracture. In this technique, the total energy functional (sum of bulk energy and surface energy) is minimized to obtain the practically feasible crack nucleation and propagation. Bourdin [23] derived a variational formulation, which was easy for numerical implementation. Most important is that a sharp crack topology is regularized over the entire domain using a scalar parameter, later known as a phase field variable [24]. To distinguish fracture in tension and compression, first effort was made by Amor et al. [25] and suggested the volumetric and deviatoric decompositions of strain energy for the variational formulation. A revolutionary work in fracture using phase field was performed by Miehe et al. [26,27]. They developed a thermodynamic consistent PFM to model brittle fracture. To avoid cracking in compression, the strain energy was decomposed on the basis of the spectral decomposition of the strain. Miehe et al. [27] also provided a staggered scheme for phase field fracture and demonstrated the robustness against anisotropic model [26]. As per Li [28] for rocks or stones, the deviatoric model [25] may predict realistic crack path whereas for more brittle materials such as concrete or glass, models based on a spectral decomposition [27] might be more suitable. Borden et al. [29] suggested a fourth-order phase field approach to simulate brittle fracture in the isogeometric framework. This model leads to higher regularity in the exact phase field solution when applied for 3-D crack growth simulation. Ambati et al. [30] proposed a hybrid model, which avoids a nonlinear stress-strain relationship in the staggered formulation, and thus leads to an enhanced computational efficiency. They also provided a review of the brittle fracture using PFM under quasi-static and dynamic conditions. This hybrid phase field method was extended by Doan et al. [31,32] to simulate the dynamic fracture in inhomogeneous and functionally graded materials. The PFM was further extended to simulate fracture in plates and shells [33,34]. Recently, the monolithic and staggered phase field schemes were implemented in the commercial finite element software (ABAQUS) for the quasi-static [35] and dynamic fracture [36] using UEL and UMAT subroutines. On similar lines, the PFM was implemented in COMSOL [37–39] as it can be easily extended to coupled field problems. The PFM was also extended to analyze fracture failure in the heterogeneous material. Nguyen et al. [40] used a phase field model to analyze the crack initiation and growth in the heterogeneous concrete material. The fracture failure of the heterogeneous anisotropic nuclear graphite was investigated by Chakraborty et al. [41]. The fracture toughness of polymer clay nano-composites was predicted using fine scale phase field model [42,43].

Some contributions were also made to improve the computational efficiency of the phase field method by utilizing an adaptive mesh refinement scheme. To this end, Mahnken [44] proposed a goal-oriented error estimation adaptive mesh refinement strategy in the finite element framework for phase field modeling of the phase transformation. Heister [45] presented a predictor-corrector strategy to achieve local mesh adaptivity for finite element based PFM to simulate the brittle fracture. On the other hand, Lee et al. [46] developed a PFM based robust and efficient mesh adaptive scheme to reduce the computational cost of large-scale 3-D structures subjected to hydraulic pressure. Zhang et al. [47] proposed a moving mesh finite element along with regularization methods for better convergence of Newton’s iteration in phase field simulation. Some local mesh refinement algorithms were also proposed in Refs. [48–50] to solve the fracture problems.

For enhancing the computational efficiency of the FEM simulations, Zhang et al. [51] proposed a multiscale finite element method (MsFEM) to solve elastic, elasto-plastic and dynamic problems in non-homogeneous media at different length scales. The MsFEM is the two scale approach where lower scale information is gathered at upper scale by using multiscale basis functions. Wu et al. [52] developed an approach, which was the coupling of MsFEM with XFEM to simulate quasi-static crack growth in the locally refined small area near the discontinuity known as the concurrent multiscale method.

The simulation of fracture failure using standard PFM requires highly dense (refined) mesh to obtain small crack regularization parameter for the representation of sharp crack in the domain. However, the use of dense mesh in the entire domain becomes computationally inefficient. To overcome this issue, recently we [53] developed an approach, which is the coupling of the hybrid PFM [30] with MsFEM [51] to obtain adaptively refined mesh near a smeared discontinuity. It was demonstrated that this coupling provides a significant saving in the computational cost. Although, the hybrid PFM is a simple phase field approach to simulate fracture, where decomposition of the strain tensor into tensile and compressive components was not considered to avoid nonlinear stress-strain relations in elasticity equilibrium equation (due to easy implementation). For an accurate modelling of complex crack growth using phase field, it is important to differentiate fracture in tension and compression.

Therefore, in the present work, anisotropic PFM [27] is coupled with MsFEM [51] to simulate complex crack growth mechanism of the composites made of highly brittle constituents. The coupling of anisotropic PFM with MsFEM is referred as multiscale phase field method (MsPFM). In anisotropic phase field formulation, the strain tensor is decomposed into tensile and compressive components based on the spectral decomposition of strain, which is quite suitable for phase field modeling of highly brittle materials [28]. The proposed MsPFM is formulated such that, it adaptively refines the mesh to simulate the complex crack growth pattern of the composites such as crack arrest, crack deflection, crack branching, crack coalescence and multiple crack nucleation. A detailed formulation of the MsPFM is provided in Section 2. During the MsPFM implementation, the domain selected for the fracture analysis is discretized in such a way that the local area in the vicinity of the smeared crack is identified and meshed with dense/refined mesh. Simultaneously, the region away from the smeared crack is meshed with MsFEM coarse elements. The dense and coarse meshes are connected using master-slave constraints defined by multiscale basis functions to get an integrated computational model. This integrated computational model will be solved for externally applied loads. The necessary details on the implementation of the MsPFM are presented in Section 3. In Section 4, some numerical experiments have been performed to demonstrate the aforementioned failure characteristics of the composite. In the first two cases, the crack growth pattern of interfacial crack in a bi-material domain is investigated for the crack deflection, and the obtained crack path is compared with the XFEM solution. A numerical example is also solved to model the crack coalescence in bi-materials. Few examples are solved to study the interaction of the matrix crack with the strong and weak fibre-matrix interface in the fibre reinforced ceramic composite. The effect of the strong and weak interface between the plies is also studied on the fracture behavior of the laminated composite.

In all simulations, the constituents of the composite are assumed to be homogeneous and isotropic. A single critical fracture toughness value is used for the individual phases (fibre/matrix/interface) in all the loading modes assuming that the fracture toughness is independent of the loading mode. Specifically, the novelty of the present work are as follows,

- A new MsPFM is proposed by coupling MsFEM [51] with the anisotropic PFM [27].
- The MsPFM considers a nonlinear stress-strain relationship in the stress equilibrium equation.
- The MsPFM is used to simulate failure of the composites considering the effect of weak and strong interfaces on the fracture toughness and peak load.
- An adaptive mesh discretization strategy is developed to handle...