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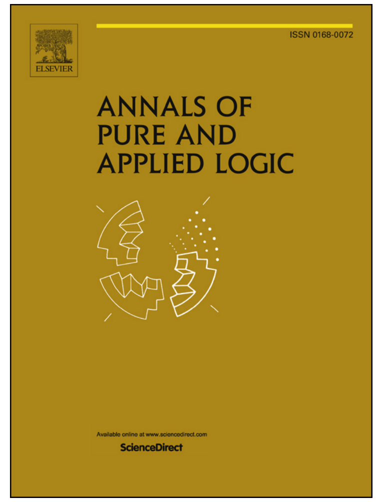

Cut-elimination for ω_1

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Cut-elimination for ω_1

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Abstract

In this paper we calibrate the strength of the soundness of a set theory $\mathsf{KP}\omega+(\Pi_1\text{-Collection})$ with the assumption that 'there exists an uncountable regular ordinal' in terms of the existence of ordinals.

Keywords: cut-elimination, uncountable regular ordinal, reflection principle
2000 MSC: 03F03, 03F05, 03F07, 03F99

1. Introduction

In [2, 3] higher set theories are analyzed proof-theoretically in terms of the operator controlled derivations, which are introduced by W. Buchholz[6]. Let I be the least weakly inaccessible cardinal. Collapsing functions $\alpha \mapsto \Psi_{\kappa,n}(\alpha) < \kappa$ are introduced for each uncountable regular cardinal $\kappa \leq I$ and $n < \omega$. $\Psi_{\kappa,n}(\alpha)$ is a first-order variant of the collapsing functions $\psi_\kappa(\alpha)$ introduced by W. Buchholz[5]. Let $\omega_k(I+1)$ denote the tower of ω with the next epsilon number $\varepsilon_{I+1} = \sup\{\omega_k(I+1) : k < \omega\}$ above I. The predicate $x = \Psi_{\kappa,n}(\alpha)$ is a Σ_{n+1}-predicate for $\alpha < \varepsilon_{I+1}$, and we see that for each $n, k < \omega$ $\mathsf{ZF}+(V=L)$ proves $\forall\alpha < \omega_k(I+1)\forall\kappa \leq I\exists x < \kappa[x = \Psi_{\kappa,n}(\alpha)]$.

Conversely the following Theorem 1.1 is shown in [3].

Theorem 1.1. *For a sentence $\exists x \in L_{\omega_1}\varphi(x)$ with a first-order formula $\varphi(x)$, if*

$$\mathsf{ZF}+(V=L) \vdash \exists x \in L_{\omega_1}\varphi(x)$$

then

$$\exists n < \omega[\mathsf{ZF}+(V=L) \vdash \exists x \in L_{\Psi_{\omega_1,n}(\omega_n(I+1))}\varphi(x)].$$