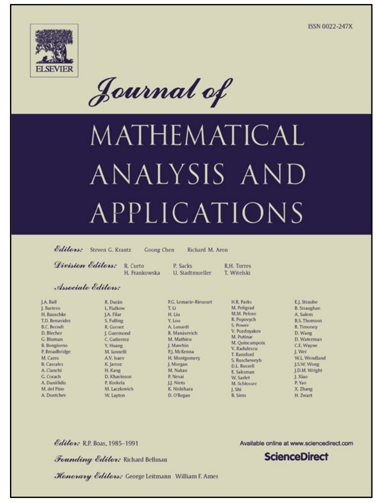

A note on Hamilton sequences of extremal Beltrami differentials

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A NOTE ON HAMILTON SEQUENCES OF EXTREMAL BELTRAMI DIFFERENTIALS

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Abstract. Let X be a hyperbolic Riemann surface and let μ be an extremal Beltrami differential on X with $\|\mu\|_\infty \in (0,1)$. It is proved that, if $\{\phi_n\}$ is a Hamilton sequence of μ, then $\{\phi_n\}$ must be a Hamilton sequence of any extremal Beltrami differential ν contained in $[\mu]$. This result proved a conjecture of the first author of this paper in 1996. This result is also a generalization of two known results.

§1. Introduction

Let X be a given hyperbolic Riemann surface and let $Belt(X)$ be the Banach space of bounded measurable Beltrami differentials $\mu = \mu(z)\overline{dz}/dz$ on X with L_∞-norm.

By $\mathcal{M}(X)$ we denote the open unit ball of $Belt(X)$, namely

$$\mathcal{M}(X) := \{\mu \in Belt(X) : \|\mu\| < 1\}.$$

For any given element μ of $\mathcal{M}(X)$, there is a quasiconformal mapping f^μ of X onto X with the Beltrami coefficient μ, which is normalized by

$$f^\mu(\pm 1) = \pm 1 \quad \text{and} \quad f^\mu(i) = i.$$

Such a quasiconformal mapping $f^\mu : X \to X$ is uniquely determined by μ.

Two elements μ and ν in $\mathcal{M}(X)$ are said to be Teichmüller equivalent to each other, denoted by $\mu \sim \nu$, if there is a conformal mapping of Φ of X onto X such that

$$(f^\nu)^{-1} \circ \Phi \circ f^\mu \;\text{ is homotopic to the identity map of } X \text{ (Mod } \partial X).$$

The Teichmüller space $T(X)$ of X is defined as the quotient space $\mathcal{M}(X)/\sim$:

$$T(X) := \{[\mu] : \mu \in \mathcal{M}(X)\},$$

where $[\mu]$ is the Teichmüller equivalence class of μ.

An element μ in $\mathcal{M}(X)$ is said to be an extremal, if

$$\|\mu\|_\infty \le \|\mu'\|_\infty, \quad \forall \mu' \in [\mu].$$

Let $\mathcal{Q}(X)$ be the Banach space of integrable holomorphic quadratic differentials $\phi = \phi(z)dz^2$ on X with L_1-norm.

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