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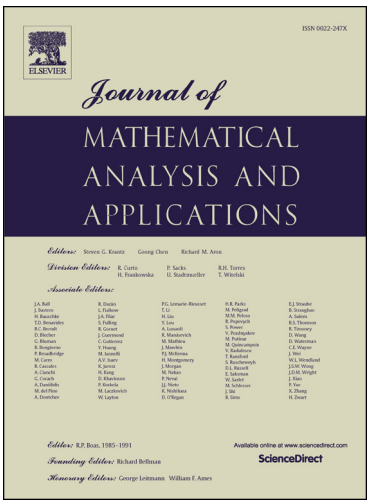

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SENSITIVITY OF ITERATED FUNCTION SYSTEMS

F. H. GHANE, E. REZAALI, AND A. SARIZADEH*

Abstract. The present work is concerned with the eqiucontinuity and sensitivity of iterated function systems (IFSs). Here, we consider more general case of IFSs, i.e. the IFSs generated by a family of relations. We generalize the concepts of transitivity, sensitivity and equicontinuity to these kinds of systems. This note investigates the relationships between these concepts. Then, several sufficient conditions for sensitivity of IFSs are presented. We introduce the notion of weak topologically exact for IFSs generated by a family of relations. It is proved that non-minimal weak topologically exact IFSs are sensitive. That yields to different examples of non-minimal sensitive systems which are not an M-system. Moreover, some interesting examples are given which provide some facts about the sensitive property of IFSs.

1. Introduction

In deterministic dynamical systems, chaos concludes all random phenomena without any stochastic factors. On the study of chaos, the concept of sensitivity is a key ingredient. In fact, sensitivity characterizes the unpredictability of chaotic phenomena, and is one of the essential conditions in various definitions of chaos. Therefore, the study on sensitivity has attracted a lot of attention from many researchers (e.g. [1, 5, 6, 8, 23, 25, 30, 31]). In 1971, Ruelle introduced the first precise definition for sensitivity [36]. Then a formulation of sensitivity was given by Guckenheimer on the study of interval maps, [24]. In 1986, Devaney [16] proposed the widely accepted definition of chaos (topological transitivity, dense periodic points and sensitivity), and emphasized the significance of sensitivity in describing dynamical systems. Afterwards, Li-Yorke sensitivity [6], n-sensitivity [39], and collective sensitivity [38] were successively proposed, and each of these concepts is used to describe the complexity of dynamical systems. For continuous self-maps of compact metric spaces, Moothathu [34] initiated a preliminary study of stronger forms of sensitivity formulated in terms of large subsets of $\mathbb{N}$. Mainly he considered syndetic sensitivity and cofinite sensitivity and proved that any syndetically transitive, non-minimal map is syndetically sensitive. Then, Liu, Liao and Wang [32] introduced some other versions of sensitivities, including thick sensitivity and thickly syndetical sensitivity. After that Wang, Yin and Yan [37] extended some of these results to semigroup actions. They presented some sufficient conditions for dynamical systems of semigroup actions to have these sensitivities.

Although the sensitivity is widely understood as the central idea in Devaney chaos, but it is implied by transitivity and density of periodic points [8]. This result was generalized in [4], changing density of periodic points to density of minimal points. In fact the authors proved

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