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Uniform asymptotics for finite-time ruin probability of a bidimensional risk model

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ABSTRACT

Consider a continuous-time bidimensional risk model with constant force of interest in which the claim sizes from the same business are heavy-tailed and upper tail asymptotically independent. We investigate two cases: one is that the two claim-number processes are arbitrarily dependent, and the other is that the two corresponding claim inter-arrival times from different lines are positively quadrant dependent. Some uniformly asymptotic formulas for finite-time ruin probability are established.

1. Introduction

Consider a bidimensional continuous-time risk model, in which an insurance company simultaneously operates two kinds of businesses. The surplus process can be described as

$$\begin{pmatrix} U_1(x_1,t) \\ U_2(x_2,t) \end{pmatrix} = \mathrm{e}^{rt} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \int_{0-}^{t} \mathrm{e}^{r(t-y)} C_1(\mathrm{d}y) \\ \int_{0-}^{t} \mathrm{e}^{r(t-y)} C_2(\mathrm{d}y) \end{pmatrix} - \begin{pmatrix} \sum_{i=1}^{N_1(t)} X_i^{(1)} \mathrm{e}^{r(t-\sigma_i^{(1)})} \\ \sum_{i=1}^{N_2(t)} X_i^{(2)} \mathrm{e}^{r(t-\sigma_i^{(2)})} \end{pmatrix}, \tag{1.1}$$

where $r \geq 0$ denotes the constant force of interest, $(x_1, x_2)^\top \geq (0,0)^\top$ the vector of initial reserve, $\{(C_1(t), C_2(t))^\top,\ t \geq 0\}$ the bidimensional premium process, $\{(X_i^{(1)}, X_i^{(2)})^\top,\ i \geq 1\}$ the sequence of claim-size vectors, and $\{(\sigma_i^{(1)}, \sigma_i^{(2)})^\top,\ i \geq 1\}$ the sequence of claim-arrival-time vectors with the corresponding bidimensional claim-number process $\{(N_1(t), N_2(t))^\top,\ t \geq 0\}$. Here, $N_k(t) = \sup\{i \geq 0 : \sigma_i^{(k)} \leq t\},\ t \geq 0,\ k = 1,2$. Let $\{(\theta_i^{(1)}, \theta_i^{(2)})^\top,\ i \geq 1\}$ be the sequence of claim-inter-arrival-time vectors. Then the claim arrival times satisfy $\sigma_i^{(k)} = \sum_{j=1}^{i} \theta_j^{(k)},\ i \geq 1,\ k = 1,2$. For each $k = 1,2$, and all

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$t \geq 0$, denote the mean function by $\lambda_k(t) = \mathbb{E}N_k(t)$ with $\lambda_k(0) = 0$ and $\lambda_k(t) < \infty$, and define the set $\Lambda_k = \{t > 0 : \lambda_k(t) > 0\} = \{t > 0 : \mathbb{P}(\sigma_1^{(k)} \leq t) > 0\}$. If we set $\underline{t}^{(k)} = \inf\{t > 0 : \mathbb{P}(\sigma_1^{(k)} \leq t) > 0\}$, then it is easy to see that $\Lambda_k = [\underline{t}^{(k)}, \infty]$ if $\mathbb{P}(\sigma_1^{(k)} = \underline{t}^{(k)}) > 0$; or $\Lambda_k = (\underline{t}^{(k)}, \infty]$ if $\mathbb{P}(\sigma_1^{(k)} = \underline{t}^{(k)}) = 0$. We denote the intersection set by $\Lambda = \Lambda_1 \cap \Lambda_2$. For each $k = 1, 2$, if $\{\theta_i^{(k)},\ i \geq 1\}$ are independent and identically distributed (i.i.d.), then $\{N_k(t),\ t \geq 0\}$ is a so-called renewal counting process.

For each $k = 1, 2$, assume that the claim sizes from the k-th line $\{X_i^{(k)},\ i \geq 1\}$ is a sequence of identically distributed nonnegative random variables (r.v.s) with a common distribution F_k; the claim inter-arrival times $\{\theta_i^{(k)},\ i \geq 1\}$ is another sequence of nonnegative r.v.s; and the premium process $\{C_k(t),\ t \geq 0\}$ is a nonnegative and nondecreasing stochastic process with $C_k(0) = 0$ and $C_k(t) < \infty$ almost surely for every $t \geq 0$. Assume further that the four sequences $\{X_i^{(k)},\ i \geq 1\},\ k = 1, 2,\ \{(N_1(t), N_2(t))^\top,\ t \geq 0\}$ and $\{(C_1(t), C_2(t))^\top,\ t \geq 0\}$ are mutually independent.

Let the time of ruin be

$$\tau_{\max}(x_1, x_2) = \inf\{t > 0 : \max\{U_1(x_1, t), U_2(x_2, t)\} < 0\},$$

with $\inf \emptyset = \infty$ by convention. Then the ruin probability within finite time $t > 0$ corresponding to the risk model (1.1) can be defined as

$$\begin{aligned}\psi(x_1, x_2; t) &= \mathbb{P}\left(\tau_{\max}(x_1, x_2) \leq t \,\middle|\, (U_1(x_1, 0), U_2(x_2, 0))^\top = (x_1, x_2)^\top\right)\\ &= \mathbb{P}\left(\bigcap_{i=1}^{2}\left(\inf_{0 \leq s \leq t} U_i(x_i, s) < 0\right) \,\middle|\, (U_1(x_1, 0), U_2(x_2, 0))^\top = (x_1, x_2)^\top\right).\end{aligned}$$

In this paper we are interested with the precisely asymptotic behavior of the finite-time ruin probability $\psi(x_1, x_2; t)$ with the uniformity in t over some subsets of Λ as $(x_1, x_2)^\top \to (\infty, \infty)^\top$.

In the past two decades, the investigation of the uniform asymptotics for the finite-time ruin probability in one-dimensional risk models has attracted a vast amount of attention due to their practical importance. In the presence of heavy-tailed claim sizes, Tang [15], Leipus and Šiaulys [6,7], Yang et al. [21] and Wang et al. [18] investigated some independent or dependent risk models with no interest force, whereas Wang et al. [17] considered the model with constant interest force and Li et al. [10], Li [8] and Yang et al. [23] further studied the models with stochastic return. Recently, more attention has been paid to multi-dimensional risk models, especially bidimensional ones. Some existing results require that the two claim-number processes are independent, but more literature is concerned on the bidimensional model with a common claim-number process. Chen et al. [2] considered an independent one with no interest force, and obtained some uniformly asymptotic formulas for finite-time ruin probability. Chen et al. [1] extended their results by allowing some certain dependence among the common inter-arrival times. Jiang et al. [4] further derived a uniform asymptotic result in which the claim-size vector and the corresponding inter-arrival time fulfill some certain dependence conditions. Some other related results can be found in [19] (whose main result is covered by Jiang et al. [4]), Li and Yang [11], Yang and Yuen [24], Yang and Yuen [25], Li [9], Yang et al. [27], Yang et al. [26] and among others. We remark that in most aforementioned works there are two kinds of extremal assumptions on the two claim-number processes: one is to assume that the two claim-number processes are mutually independent; the other is that they are same, that is, the two kinds of businesses share a common claim-number process. However, some dependence should exist in the two processes due to practical relevance. For instance, in the businesses of the car insurance and the medical insurance by car accidents, a car accident may cause one claim for vehicle damage immediately and more than one medical claims for injures of both drivers and passengers in the subsequent periods. Thus, the two claim-number processes are neither independent nor same, and it is hard to describe the dependence in detail. Based on this, one consideration of this paper is to allow (arbitrary) dependence between the two claim-number processes, which is motivated by a recent work of Yang and Li [20]. Another consideration is to allow some