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Wang's Harnack inequalities for space–time white noises driven SPDEs with two reflecting walls and their applications

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ABSTRACT

In this paper, we establish Wang's Harnack inequalities for Gaussian space–time white noises driven the stochastic partial differential equation with double reflecting walls, which is of the infinite dimensional Skorokhod equation. We first establish both the Harnack inequality with power and the log-Harnack inequality for the special case of additive noises by the coupling approach. Then we investigate the log-Harnack inequality for the Markov semigroup associated with the reflected SPDE driven by multiplicative noises using the penalization method and the comparison principle for SPDEs. As their applications, we study the strong Feller property, uniqueness of invariant measures, the entropy-cost inequality, and some other important properties of the transition density.

1. Introduction

The main purpose of this paper is to investigate Wang's Harnack inequalities for the transition semigroup associated with the following stochastic partial differential equation (SPDE for abbreviation) with two reflecting walls driven by a Gaussian space–time white noise:

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) = \dfrac{1}{2}\dfrac{\partial^2 u}{\partial x^2}(t,x) + b(u(t,x)) + \sigma(u(t,x))\dot{W}(t,x) \\ \qquad\qquad + \eta(dtdx) - \zeta(dtdx),\ t>0,\ x\in(0,1), \\ \xi_1(x) \le u(t,x) \le \xi_2(x),\ t\ge 0, x\in[0,1], \\ u(t,0) = u(t,1) = 0,\ t\ge 0, \\ u(0,x) = h(x),\ x\in[0,1], \end{cases} \tag{1.1}$$

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where the homogeneous Dirichlet boundary conditions are considered, the initial value h is a measurable function on $[0, 1]$ satisfying $h(0) = h(1) = 0$, ξ_1 and ξ_2 are two continuous functions on $[0, 1]$, which are called the reflecting walls, η and ζ are two positive random measures on $[0, +\infty) \times [0, 1]$, the coefficients b and σ are measurable real-valued functions defined on $\mathbb{R}$, the noise term $\{\dot{W}(t, x) : t \geq 0, x \in [0, 1]\}$ denotes the Gaussian space–time white noise defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a filtration $(\mathcal{F}_t)_{t\geq 0}$. For detailed assumptions and the precise mathematical meaning of the SPDE (1.1), see Section 2.

The reflected SPDE (1.1) is an important example of random parabolic obstacle problems. This kind of problems is also regarded as an infinite-dimensional Skorokhod problem. Hence the study of the SPDEs with reflections attracts many authors attention after the significant work [13].

Let us first review some researches on reflected SPDEs. We begin with SPDEs with one reflecting wall. It is first introduced by D. Nualart and E. Pardoux [13] in 1992 for the case of additive noises, that is, σ is a constant. In that paper, the authors mainly studied the existence and uniqueness of solution of the reflected SPDE. In 1993, C. Donati-Martin and E. Pardoux [8] generalized [13] to the case of the reflected SPDE driven by multiplicative noises and they constructed successfully a minimum solution.

At present, many interesting properties of the solutions have been actively studied. For example, reversible measures [9,32], hitting properties [3], Harnack inequality with power [34] were studied for the reflected SPDE driven by additive noises. It is also well-known that reflected SPDEs have been successfully applied to characterize the fluctuation of a $\nabla\phi$-interface model on a hard wall by T. Funaki and S. Olla [9] in 2001. They showed that the fluctuation of such model weakly converges to the stationary solution of the reflected SPDE driven by additive noises. It is a very interesting and important application of reflected SPDEs, and then it furthermore promotes the research of reflected SPDEs.

On the other hand, for the general multiplicative noises, the study of it is harder than the case of additive noises and rare properties have been studied. In fact, the uniqueness of its solutions is left open in [8] for a very long time and at last it is successfully solved in [30] until 2009. Recently, thanks to the contribution of [30], the strong Feller property [34], the large deviation principle [30] and the log-Harnack inequality [29] relative to the reflected SPDE with one reflection driven by multiplicative noises have been investigated respectively. We refer the readers to the monograph [33] by L. Zambotti in 2017 and references therein for more information on the study of SPDEs with one reflecting wall.

From now on, let us turn to the reflected SPDE with two reflecting walls, which is the main object of this paper. This kind of reflected SPDEs is first studied by Y. Otobe [14] based on the main methodology in [13] and the existence and uniqueness of the solutions of the SPDE (1.1) driven by additive noises are established. For the general case of the SPDE (1.1) driven by multiplicative noises is investigated by T.-S. Zhang and J. Yang [36], which is harder than the case of additive noises. However, there are only a few properties of the solution to the SPDE (1.1) have been studied, for example, the large deviation principle [35], existence and uniqueness of invariant measures [31] for periodic boundary conditions.

The main purpose of this paper is to establish Wang's Harnack inequalities, which are also called the dimension-free Harnack inequalities in many literatures and are different from the classical ones, for the transition semigroup associated with the Markov process determined by the solution $u(t, x)$ of the reflected SPDE (1.1), which are helpful for us to understand the SPDE (1.1) deeply. Wang's Harnack inequalities are initially introduced in [21] by F.-Y. Wang for a diffusion process on Riemannian manifolds and then become very effective to the study of various important properties of diffusion semigroups, such as hypercontractivity, ultracontractivity [17,26], strong Feller property, estimates on the heat kernels [22] and Varadhan type small time asymptotics [1,34].

Recently, Wang's Harnack inequalities have also been actively studied and applied to the transition semigroup associated with SPDEs. For example, see [4,11,15,19,22,25,34] for the study of the Harnack inequality with power and respectively to [18,27,28] for the study of the log-Harnack inequality. Here we remark that the log-Harnack inequality relative to usual SPDEs driven by multiplicative noises without