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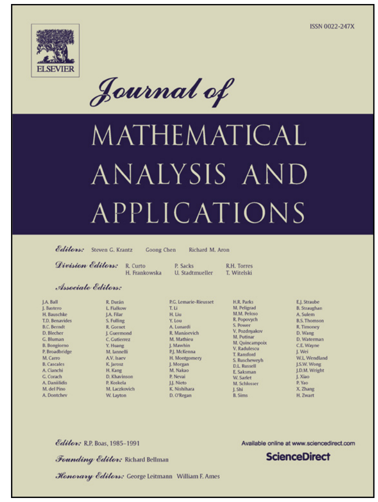

Stochastic solutions for space-time fractional evolution equations on bounded domain

Lorenzo Toniazzi

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ACCEPTED MANUSCRIPT

STOCHASTIC SOLUTIONS FOR SPACE-TIME FRACTIONAL EVOLUTION EQUATIONS ON BOUNDED DOMAIN

Lorenzo Toniazzi

Abstract. Space-time fractional evolution equations are a powerful tool to model diffusion displaying space-time heterogeneity. We prove existence, uniqueness and stochastic representation of classical solutions for an extension of Caputo evolution equations featuring time-nonlocal initial conditions. We discuss the interpretation of the new stochastic representation. As part of the proof a new result about inhomogeneous Caputo evolution equations is proven.

1. Introduction

It is a classical result that the solution to the standard heat equation $\partial_t u = \Delta u$, $u(0) = \phi_0$ allows the stochastic representation $u(t, x) = \mathbf{E}[\phi_0(X^{x,2}(t))]$, where $X^{x,2}$ is a Brownian motion started at $x \in \mathbb{R}^d$. Space-time fractional evolution equations (EEs) extend the heat equation by introducing space-time heterogeneity. This often is done by considering the Caputo EE $D_0^\beta u = -(-\Delta)^{\frac{\alpha}{2}} u$, where one substitutes the local operators ∂_t and Δ with fractional analogues. Respectively, the Caputo derivative $D_0^\beta u(t) = c_\beta \int_0^t u'(r)(t-r)^{-\beta}\, dr$ and the fractional Laplacian $(-\Delta)^{\frac{\alpha}{2}} u(x) = \mathcal{F}^{-1}(|\xi|^\alpha \mathcal{F}u(\xi))(x)$, where $\beta \in (0, 1)$, $\alpha \in (0, 2)$, $c_\beta = \Gamma(1-\beta)^{-1}$ and $\mathcal{F}$ is the Fourier transform (for standard references see [21, 14]). It is well known that the fundamental solution to the Caputo EE is the law of the non-Markovian anomalous diffusion $Y^x(t) = X^{x,\alpha}(\tau_0(t))$ (see, e.g., [39]). Here $X^{x,\alpha}$ is the rotationally symmetric α-stable Lévy process started at $x \in \mathbb{R}^d$ and $\tau_0(t)$ is the inverse process of the β-stable subordinator $X^\beta(t)$. The density of this beautiful formula was first observed in [43]. The time change interpretation first appeared in [35, 38], based on [5]. The process Y^x displays space-heterogeneity due to the jump nature of $X^{x,\alpha}$. Also time-heterogeneity features in Y^x, as the time change $t \mapsto \tau_0(t)$ is constant precisely when the subordinator $t \mapsto X^\beta(t)$ jumps, so that $t \mapsto Y^x(t)$ is trapped on such time intervals. This interesting trapping phenomenon leads to the process Y^x spreading at a slower rate than $X^{x,\alpha}$. Indeed, in the physics literature the anomalous diffusion Y^x is often referred

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