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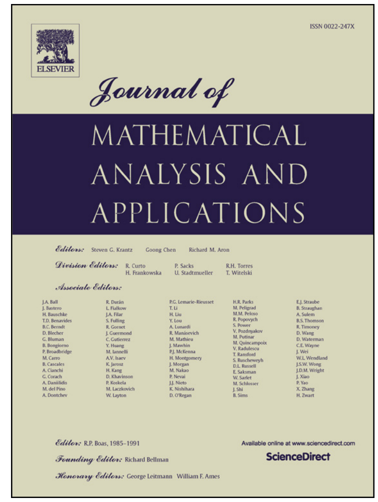

Zero distribution of some shift polynomials

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ZERO DISTRIBUTION OF SOME SHIFT POLYNOMIALS

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ABSTRACT. Given an entire function f of finite order ρ, let $g(f) := \sum_{j=1}^{k} b_j(z)f(z+c_j)$ be a shift polynomial of f with small meromorphic coefficients b_j in the sense of $O(r^{\lambda+\varepsilon}) + S(r,f)$, $\lambda < \rho$. Provided α, β, b_0 are similar small meromorphic functions, we consider zero distribution of $f^n(g(f))^s - b_0$, resp. of $g(f) - \alpha f^n - \beta$.

1. INTRODUCTION

During the last decade, Nevanlinna theory for differences (and shifts) of meromorphic functions has been a topic of substantial interest, see e.g. [6], to some extent due to the extensive investigations related to discrete Painlevé equations. The reader is invited to consult the survey [3] on this topic.

In this paper, we assume familiarity with the key notions and results in the Nevanlinna theory [7]. In addition to the main theorems in Nevanlinna theory, we frequently need to apply the notion of exponent of convergence $\lambda(f)$ for zeros of f, the Clunie reasoning, see [8], and the notion of small functions with respect to a given meromorphic function. Unless otherwise specified, a meromorphic function α is said to be small in this paper, relative to a given meromorphic function f of finite order ρ, if for any $\varepsilon > 0$, and for some $\lambda < \rho$, $T(r,\alpha) = O(r^{\lambda+\varepsilon}) + S(r,f)$ outside of a possible exceptional set of finite logarithmic measure.

The starting point to us is the following simple result proved in [9], see Theorem 2 therein:

Theorem A. *Let $f(z)$ be a transcendental entire function of finite order, and c be a non-zero complex constant. Then for $n \geq 2$, $f(z)^n f(z+c)$ assumes every non-zero value $a \in \mathbb{C}$ infinitely often.*

This paper prompted a number of related investigations during the last ten years. As to these developments, we refer to [1], Chapter 4.1 and the references given therein. Typical results in this direction are: (1) Provided $p(z)$ is a non-vanishing polynomial, then $f(z)^n f(z+c) - p(z)$, resp. $f(z)^n(f(z+c) - f(z)) - p(z)$, admits infinitely many zeros, see [1], Theorem 4.1.3 and Theorem 4.1.4. (2) Analyzing zeros, resp. a-points, $a \neq 0$, for $f(z)^n(f(z+c) - f(z)), n \geq 1$, see [1], Chapter 4.1.

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