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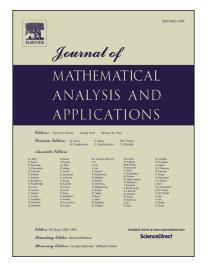
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### ACCEPTED MANUSCRIPT

#### ZERO DISTRIBUTION OF SOME SHIFT POLYNOMIALS

#### ILPO LAINE

ABSTRACT. Given an entire function f of finite order  $\rho$ , let  $g(f) := \sum_{j=1}^{k} b_j(z) f(z+c_j)$  be a shift polynomial of f with small meromorphic coefficients  $b_j$  in the sense of  $O(r^{\lambda+\varepsilon}) + S(r, f), \lambda < \rho$ . Provided  $\alpha, \beta, b_0$  are similar small meromorphic functions, we consider zero distribution of  $f^n(g(f))^s - b_0$ , resp. of  $g(f) - \alpha f^n - \beta$ .

#### 1. INTRODUCTION

During the last decade, Nevanlinna theory for differences (and shifts) of meromorphic functions has been a topic of substantial interest, see e.g. [6], to some extent due to the extensive investigations related to discrete Painlevé equations. The reader is invited to consult the survey [3] on this topic.

In this paper, we assume familiarity with the key notions and results in the Nevanlinna theory [7]. In addition to the main theorems in Nevanlinna theory, we frequently need to apply the notion of exponent of convergence  $\lambda(f)$  for zeros of f, the Clunie reasoning, see [8], and the notion of small functions with respect to a given meromorphic function. Unless otherwise specified, a meromorphic function  $\alpha$  is said to be small in this paper, relative to a given meromorphic function f of finite order  $\rho$ , if for any  $\varepsilon > 0$ , and for some  $\lambda < \rho$ ,  $T(r, \alpha) = O(r^{\lambda + \varepsilon}) + S(r, f)$  outside of a possible exceptional set of finite logarithmic measure.

The starting point to us is the following simple result proved in [9], see Theorem 2 therein:

**Theorem A.** Let f(z) be a transcendental entire function of finite order, and c be a non-zero complex constant. Then for  $n \ge 2$ ,  $f(z)^n f(z+c)$  assumes every non-zero value  $a \in \mathbb{C}$  infinitely often.

This paper prompted a number of related investigations during the last ten years. As to these developments, we refer to [1], Chapter 4.1 and the references given therein. Typical results in this direction are: (1) Provided p(z) is a non-vanishing polynomial, then  $f(z)^n f(z+c) - p(z)$ , resp.  $f(z)^n (f(z+c) - f(z)) - p(z)$ , admits infinitely many zeros, see [1], Theorem 4.1.3 and Theorem 4.1.4. (2) Analyzing zeros, resp. *a*-points,  $a \neq 0$ , for  $f(z)^n (f(z+c) - f(z)), n \geq 1$ , see [1], Chapter 4.1.

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