Accepted Manuscript

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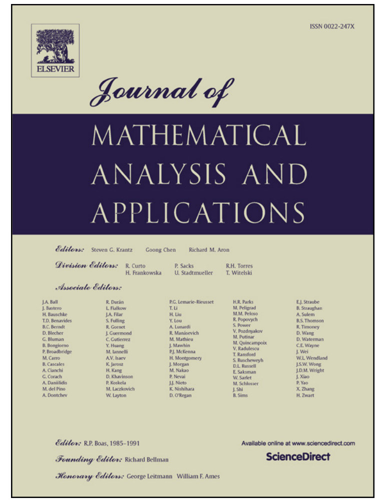

PII: S0022-247X(18)30786-8
DOI: https://doi.org/10.1016/j.jmaa.2018.09.037
Reference: YJMAA 22557

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 4 March 2018

Components reduction regularity results for the Navier-Stokes equations in general dimensions

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Abstract

We consider the Cauchy problem of the Navier-Stokes equations in arbitrary dimensions, and establish several new components reduction regularity criteria.

Keywords: Navier-Stokes equations, blow-up criterion, vorticity

2010 MSC: 35B65, 35Q35, 76D03

1. Introduction

In this paper, we study the Navier-Stokes system in $\mathbb{R}^N$ $(N \geqslant 3)$:

$$\begin{cases} \partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \Delta\boldsymbol{u} + \nabla\pi = \boldsymbol{0}, \\ \nabla\cdot\boldsymbol{u} = 0, \\ \boldsymbol{u}|_{t=0} = \boldsymbol{u}_0, \end{cases} \tag{1}$$

where $\boldsymbol{u}(t,x) = (u_1, u_2, \cdots, u_N)$ is the fluid velocity field; $\pi(t,x)$ is a scalar pressure, playing the role of Lagrange multiplier associated with the isochoricity constraint $\nabla\cdot\boldsymbol{u} = 0$; $\boldsymbol{u}_0(x)$ is the prescribed initial data satisfying the compatibility condition $\nabla\cdot\boldsymbol{u}_0 = 0$. Here, $(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = \sum_{i=1}^N u_i\partial_i\boldsymbol{u}$ is convection term with $\partial_i = \frac{\partial}{\partial x_i}$; $\Delta\boldsymbol{u} = \sum_{i=1}^N \partial_i^2\boldsymbol{u}$ is the Laplacian of the velocity, representing the diffusion effect of the fluid.

It is well-known that for initial data $\boldsymbol{u}_0 \in W^{s,p}(\mathbb{R}^N)$ with $2 \leqslant p < \infty,\ s > \frac{N}{p} - 1$, (1) possesses a unique local strong solution

$$\boldsymbol{u} \in C([0,T); W^{s,p}(\mathbb{R}^N)) \cap C^1((0,T); W^{s,p}(\mathbb{R}^N)) \cap C((0,T); W^{s+2,p}(\mathbb{R}^N)) \equiv CL_{s,p}(0,T), \tag{2}$$

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