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Strong accessibility and integral manifolds of the continuous-time nonlinear control systems

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ABSTRACT

The paper develops the algebraic formalism that is based on the language of ideals and modules associated with the analytic control system given by the ordinary differential equations. Using the Nagano theorem we show how the integral manifolds of Lie algebra of the considered system can be determined by the generators of some ideal of the ring of germs of analytic functions that is invariant with respect to germs of vector fields from the Lie algebra associated with the system. Additionally, basing on the language of ideals and modules the strong accessibility problem is studied. The singular points where the rank of (co)distributions associated to the system is different than at points from their neighbourhoods are considered. Using the germs of one-forms associated with the generators of the ideal one can define the integrable codistribution that in general is not analytic.

1. Introduction

In [8,9,7,16,6] some properties like generic linearizability, accessibility and strong accessibility have been studied. A property is generic if it holds on some open and dense subsets of the state space. Thus these studies have been restricted to regular points where the dimension of the (co)distribution is constant by removing from the state space some isolated points that were singular in the sense that the dimensions of (co)distributions are different than at the points from their neighbourhoods. In this paper we study the system's strong accessibility from all points in the state space, so also from the singular points. The strong accessibility from the point x_0 is defined here by using the idea of reachable states from x_0 while in [8,9,16] the concept of autonomous elements is used and it corresponds to the generic accessibility of control systems defined on homogeneous time scales. From the geometric point of view the properties of the strong accessibility from a point x_0 of the state space $\mathcal{X} \subseteq \mathbb{R}^n$ can be characterized by the fact that

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the strong accessibility distributions associated to the control systems have dimension n at x_0, see [22]. Autonomous elements can be computed by using the space $\mathcal{H}_\infty$, see for instance [13]. Then the condition $\mathcal{H}_\infty = \{0\}$ is equivalent to the strong accessibility of the system at the points belonging to the open and dense subspace of the state space. Unfortunately, $\mathcal{H}_\infty = \{0\}$ does not give the points from which the system is strongly accessible. In the paper we develop the algebraic approach to characterize the local strong accessibility. We reduce our consideration to strong accessibility at a point since it can be viewed as a generalization of controllability of linear control systems. In the case when the system is not strongly accessible the autonomous element can be used to describe the integral manifolds of Lie algebras of vector fields related with the considered systems at some points from the state space. Then not all autonomous elements obtained from $\mathcal{H}_\infty$ are used to determine the integral manifolds related with the system. One of the reasons is that studying differentials instead of functions we loose the local properties associated to the points and for example we have $\mathrm{d}(x - x_0) = \mathrm{d}x$ for the fixed x_0. Therefore to check the strong accessibility property at a point x_0 from the state space $\mathcal{X}$ we introduce modules $(\mathcal{P}_\infty)_{x_0}$ and $(H_\infty)_{x_0}$ that contain the germs of one-forms. In the case when the point x_0 belongs to the open and dense subspace of the state space modules $(H_\infty)_{x_0}$ can be used to find the differentials of the germs of autonomous elements for the system.

In the paper we introduce the ideals (in the ring of germs of analytic functions) whose generators are used to determine integral manifolds of Lie algebras of vector fields related with the considered systems. We show that zero set-germ of the ideal $(I_\infty)_{x_0}$, defined in Section 3.1, is the set-germ of the integral manifold of the Lie algebra associated to the system. In the proof of this fact Nagano Theorem is used. Then the obtained manifolds are related with the points, so the local properties of the systems are important and it is the reason of using the concept of germs of functions, one-forms and vector fields, see for instance [14]. Additionally, we study the strong accessibility problem for nonlinear analytic systems using the algebraic formalism that is based on the language of ideals and modules spanned by one-forms. We are interested in all points from the state space $\mathcal{X}$, so the singular points where the rank of (co)distributions associated to the system is different than at the points from their neighbourhoods are also considered. Since the integrable codistribution defined by the germs of one-forms associated to the generators of the introduced ideal $(I_\infty)_{x_0}$ can be used to check only a necessary condition for the strong accessibility at x_0, we define module $(H_\infty)_{x_0}$ that allows to give a necessary and sufficient condition for the strong accessibility at x_0.

The paper is organized as follows. In Section 2 the definitions of system's strong accessibility and its geometric characterization are recalled. Additionally, the motivating example is presented. In Section 3 the properties of the ideals associated to the systems are given. Section 4 shows how the sets of zeros of the introduced ideal $(I_\infty)_{x_0}$ determine the integral manifolds of Lie algebra of vector fields associated to the systems. Then we prove that the condition $(I_\infty)_{x_0} = \{0\}$ is a necessary condition for the strong accessibility from the point x_0. Section 5 gives additional conditions for strong accessibility (from the points of the state space) in the language of germs of one-forms. We prove that the subspaces corresponding to introduced modules are the annihilator of the linear spaces corresponding to the vector fields associated with the considered systems. Moreover, we show that the differentials of the generators of the ideal $(I_\infty)_{x_0}$ are used to define the module over the ring of germs (at x_0) of analytic functions. This module corresponds to an integrable codistribution, which in general is not analytic. Nextly, we present an example that illustrates our results.

In the paper we will study the local behaviour of the control systems and it is the reason we use the idea of germs that are the natural language for describing the local properties of the systems. In Appendices we provide necessary information on the germs of functions, sets, vector fields and one-forms that are used in the paper.