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SCHATTEN PROPERTIES, NUCLEARITY AND MINIMALITY OF PHASE SHIFT INVARIANT SPACES

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Abstract. We extend Feichtinger's minimality property on the smallest non-trivial time-frequency shift invariant Banach space, to the quasi-Banach case. Analogous properties are deduced for certain matrix spaces.

We use these results to prove that the pseudo-differential operator $\operatorname{Op}(a)$ is a Schatten-q operator from M^∞ to M^p and r-nuclear operator from M^∞ to M^r when $a \in M^r$ for suitable p, q and r in $(0,\infty]$.

0. Introduction

A remarkable property of the weighted Feichtinger algebra $M^1_{(v)}(\mathbf{R}^d)$ is the minimality among non-trivial Banach spaces which are invariant under time-frequency shifts with respect to the submultiplicative weight v. (See Section 1 for notations.) More precisely, let $\mathscr B$ be a Banach space which is embedded in the space of tempered distributions on $\mathbf{R}^d$ (or, more generally, in the set $\Sigma_1'(\mathbf{R}^d)$ of Gelfand-Shilov distributions of Beurling type of order 1 on $\mathbf{R}^d$). If $\mathscr B$ is invariant under time-frequency shifts,

$$\|e^{i\langle\,\cdot\,,\xi\rangle}f(\,\cdot\,-x)\|_{\mathscr B} \lesssim \|f\|_{\mathscr B}\, v(x,\xi), \qquad f\in\mathscr B,\ x,\xi\in\mathbf{R}^d, \tag{0.1}$$

and

$$\mathscr B \bigcap M^1_{(v)}(\mathbf{R}^d) \neq \{0\},$$

then $M^1_{(v)}(\mathbf{R}^d)\subseteq\mathscr B$. In fact, the search of non-trivial smallest translation and modulation invariant Banach space led Feichtinger to the discovery of $M^1(\mathbf{R}^d)$ (cf. [17]). Afterwards, Bonsall found in [5] a slightly different proof which can also be found in Section 12.1 in [25].

Several properties and ideas in [17] were further developed in [18], where Feichtinger introduced the (classical) modulation spaces, a family of time-frequency shift invariant function and distribution spaces, $M^{p,q}_{(\omega)}$, parameterised by the Lebesgue parameters $p,q\in[1,\infty]$ and suitable weights ω. The set $M^{p,q}_{(\omega)}$ is a Banach space, and is defined by

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