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SCHATTEN PROPERTIES, NUCLEARITY AND MINIMALITY OF PHASE SHIFT INVARIANT SPACES

JOACHIM TOFT

ABSTRACT. We extend Feichtinger's minimality property on the smallest non-trivial time-frequency shift invariant Banach space, to the quasi-Banach case. Analogous properties are deduced for certain matrix spaces.

We use these results to prove that the pseudo-differential operator $\text{Op}(a)$ is a Schatten- q operator from M^∞ to M^p and r -nuclear operator from M^∞ to M^r when $a \in M^r$ for suitable p, q and r in $(0, \infty]$.

0. INTRODUCTION

A remarkable property of the weighted Feichtinger algebra $M_{(v)}^1(\mathbf{R}^d)$ is the minimality among non-trivial Banach spaces which are invariant under time-frequency shifts with respect to the submultiplicative weight v . (See Section 1 for notations.) More precisely, let \mathcal{B} be a Banach space which is embedded in the space of tempered distributions on \mathbf{R}^d (or, more generally, in the set $\Sigma'_1(\mathbf{R}^d)$ of Gelfand-Shilov distributions of Beurling type of order 1 on \mathbf{R}^d). If \mathcal{B} is invariant under time-frequency shifts,

$$\|e^{i\langle \cdot, \xi \rangle} f(\cdot - x)\|_{\mathcal{B}} \lesssim \|f\|_{\mathcal{B}} v(x, \xi), \quad f \in \mathcal{B}, \quad x, \xi \in \mathbf{R}^d, \quad (0.1)$$

and

$$\mathcal{B} \cap M_{(v)}^1(\mathbf{R}^d) \neq \{0\},$$

then $M_{(v)}^1(\mathbf{R}^d) \subseteq \mathcal{B}$. In fact, the search of non-trivial smallest translation and modulation invariant Banach space led Feichtinger to the discovery of $M^1(\mathbf{R}^d)$ (cf. [17]). Afterwards, Bonsall found in [5] a slightly different proof which can also be found in Section 12.1 in [25].

Several properties and ideas in [17] were further developed in [18], where Feichtinger introduced the (classical) modulation spaces, a family of time-frequency shift invariant function and distribution spaces, $M_{(\omega)}^{p,q}$, parameterised by the Lebesgue parameters $p, q \in [1, \infty]$ and suitable weights ω . The set $M_{(\omega)}^{p,q}$ is a Banach space, and is defined by

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