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Time-dependent global attractor for extensible Berger equation

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we investigate the asymptotic behavior of the nonautonomous Berger equation

$$\varepsilon(t)u_{tt} + \Delta^2 u - (Q + \int_\Omega |\nabla u|^2 dx)\Delta u + g(u_t) + \varphi(u) = f, t > \tau,$$

on a bounded smooth domain $\Omega \subset \mathbb{R}^N$ with hinged boundary condition, where $\varepsilon(t)$ is a decreasing function vanishing at infinity. Under suitable assumptions, we establish an invariant time-dependent global attractor within the theory of process on time-dependent space.

1. Introduction

This paper is motivated by the Berger plate model [11,13]

$$\varepsilon u_{tt} + \Delta^2 u - (Q + \int_\Omega |\nabla u|^2 dx)\Delta u = p(u, u_t, x) \tag{1.1}$$

introduced by Berger in [3] as a simplification of the von Karman plate equation which describes large deflection of a plate in the two-dimensional case, where the parameter Q describes in-plane forces applied to the plate and the function p represents transverse loads which may depend on the displacement u and the velocity u_t.

When $p \equiv 0$ in (1.1), and in the one-dimensional case, the corresponding equation had been proposed by Woinowsky-Krieger [33] as a model of the transverse defection of an extensible beam as follows:

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$$\frac{\partial^2 u}{\partial t^2} + \frac{EI}{\varrho}\frac{\partial^4 u}{\partial x^4} - (\frac{H}{\varrho} + \frac{EA}{2\varrho l}\int_0^l [\frac{\partial u(\xi,t)}{\partial \xi}]^2 d\xi)\frac{\partial^2 u}{\partial x^2} = 0$$

where E is the Young's modulus, I is the cross-sectional moment of inertia, ϱ is the density, H is the tension in the rest position, l is the length of the beam in the rest position, and A is the cross-sectional area.

For more physical interpretations on extensible beam (or plate) models, we refer the reader to Eisley [20] and Ball [1], while related experimental results can be found in Burgreen [6].

With respect to the mathematical analysis of the properties of existence, stability and decay of global solutions to model (1.1), see for instance the pioneering papers by Ball [2], Dickey [16], Medeiros [26], Brito [5].

Concerning the long-time behavior of dynamical system generated by equation (1.1), there is a considerable literature that takes into account different kinds of dissipation assumptions. Some classical results are briefly reviewed as follows. Eden and Milani [19] established the existence of exponential attractor by taking $p = -u_t + h(x)$. Biazutti and Crippa [4] and Coti Zelati [15] showed the existence of global and exponential attractors by assuming $p = -\kappa(-\Delta)^\theta u_t + h(x), \kappa > 0, 0 < \theta \le 1$, the latter author also considered the continuity of the corresponding singularly perturbed system. Kouémou Patcheu [24] proved the global existence of equation (1.1) via Faedo–Galerkin method and described the asymptotic behavior of solutions with nonlinear damping $p = g(u_t)$. Recently, Yang and Wang [34] considered uniform attractor with $p = g(u_t) + f(x)$.

The aforementioned papers do not take into account the source term or nonlinear term which appears commonly to describe various self-interactions in evolution equations. Our goal is to analyze the long-term dynamics of equation (1.1) taking into consideration nonlinear damping, and with a source term and an external force, i.e. $p = -\varphi(u) - g(u_t) + f(x)$. As far as we know, in this direction, Cavalcanti et al. [7] first proved the existence by means of the Fixed Point Theorem and the stability of global solutions, based on [7], Ma and Narciso [25] investigated the existence of a global attractor. Recently, Jorge Silva and Narciso [22] studied well-posedness by the Faedo–Galerkin method and long-time dynamics for a more general system which contains a nonlocal nonlinear damping term.

All the above-mentioned papers dealt with problem (1.1) in the case where $\varepsilon(t) = 1$ is constant, the long-term behavior of equation (1.1) can be discussed in the classical framework of semigroup theory. In this paper, we aim to study problem (1.1) when ε is a function of time. More precisely, we study the long-term behavior of the following Berger equation

$$\varepsilon(t)u_{tt} + \Delta^2 u - (Q + \int_\Omega |\nabla u|^2 dx)\Delta u + g(u_t) + \varphi(u) = f, \quad t > \tau, \tag{1.2}$$

with hinged boundary conditions described by

$$u\,|_{\partial\Omega} = \Delta u\,|_{\partial\Omega} = 0, \tag{1.3}$$

and initial conditions

$$u(x,\tau) = u_0, u_t(x,\tau) = u_1. \tag{1.4}$$

Here $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, $u = u(x,t) : \Omega \times [\tau,\infty) \to \mathbb{R}$ is unknown function and $u_0, u_1 : \Omega \to \mathbb{R}$ are assigned data, Q is a constant unrestricted in sign and $f \in L^2(\Omega)$ is independent of time. The assumptions on ε, f, g will be specified in Section 2.

Compared with the aforementioned papers, the new ingredient in this paper is that the coefficient of the differential operator depends explicit on time, more precisely, we assume that ε is a (positive) decreasing