Appl. Comput. Harmon. Anal. ••• (••••) •••–•••

Contents lists available at ScienceDirect

Applied and Computational Harmonic Analysis

www.elsevier.com/locate/acha

Letter to the Editor

Extension principles for affine dual frames in reducing subspaces ☆

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ARTICLE INFO

Article history:
Received 25 September 2015
Received in revised form 1 October 2017
Accepted 11 November 2017
Available online xxxx
Communicated by Bin Han

MSC:
42C40
42C15

Keywords:
Refinable function
Affine frame
(Non)homogeneous affine dual frame
Extension principle

ABSTRACT

Mixed Oblique Extension Principles (MOEP) provide an important method to construct affine dual frames from refinable functions. This paper addresses MOEP under the setting of reducing subspaces of $L^2(\mathbb{R}^d)$. We obtain an MOEP for (non)homogeneous affine dual frames and (non)homogeneous affine Parseval frames.

1. Introduction

Extension principles provide a method for constructing affine (dual) frames from refinable functions. Among them are the Unitary Extension Principle (UEP), the Oblique Extension Principle (OEP) and the Mixed Oblique Extension Principle (MOEP). Early works on them dated back to [1–12] and references therein. Subsequently the study of Extension Principles has interested many researchers [13–25]. Observe that all above works mainly focus on homogeneous affine (dual) frames. In applications, fast wavelet transforms are our main concern, and nonhomogeneous affine (dual) frames derived from refinable functions have fast wavelet algorithms. Han in [19,26–28], Romero, Alexander, Baid, Jain and Papadakis in [29] studied nonhomogeneous affine (dual) frames. In particular, Han in [19] provided a comprehensive study on connecting homogeneous wavelets and framelets to nonhomogeneous ones with the refinable structure. This paper addresses MOEP for (non)homogeneous affine dual frames. For generality, we work under the setting

☆ The project is partially supported by the National Natural Science Foundation of China (Grant No. 11271037).

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https://doi.org/10.1016/j.acha.2017.11.006
1063-5203/

of reducing subspaces. For the details of reducing subspace affine frames, readers can refer to [7,22,23,25,30–36]. Our main contribution is to establish an MOEP under general assumptions.

A countable sequence $\{e_i\}_{i\in\mathcal{I}}$ in a separable Hilbert space $\mathcal{H}$ is called a *Bessel sequence* in $\mathcal{H}$ if there exists $C > 0$ such that

$$\sum_{i\in\mathcal{I}} |\langle f, e_i\rangle|^2 \leq C\|f\|^2 \text{ for } f \in \mathcal{H}, \tag{1.1}$$

where C is called a *Bessel bound*; it is called a *frame* for $\mathcal{H}$ if there exist constants $0 < C_1 \leq C_2 < \infty$ such that

$$C_1\|f\|^2 \leq \sum_{i\in\mathcal{I}} |\langle f, e_i\rangle|^2 \leq C_2\|f\|^2 \text{ for } f \in \mathcal{H}, \tag{1.2}$$

where C_1, C_2 are called *frame bounds*. In particular, $\{e_i\}_{i\in\mathcal{I}}$ is called a *tight frame* (*Parseval frame*) if $C_1 = C_2$ ($C_1 = C_2 = 1$) in (1.2). Given a frame $\{e_i\}_{i\in\mathcal{I}}$ for $\mathcal{H}$, a sequence $\{\tilde{e}_i\}_{i\in\mathcal{I}}$ is called a *dual* of $\{e_i\}_{i\in\mathcal{I}}$ if it is a frame such that

$$f = \sum_{i\in\mathcal{I}} \langle f, \tilde{e}_i\rangle e_i \text{ for } f \in \mathcal{H}. \tag{1.3}$$

It is easy to check that $\{e_i\}_{i\in\mathcal{I}}$ is also a dual of $\{\tilde{e}_i\}_{i\in\mathcal{I}}$ if $\{\tilde{e}_i\}_{i\in\mathcal{I}}$ is a dual of $\{e_i\}_{i\in\mathcal{I}}$. So, in this case, we also say $(\{e_i\}_{i\in\mathcal{I}}, \{\tilde{e}_i\}_{i\in\mathcal{I}})$ is a *pair of dual frames*. It is well-known that $(\{e_i\}_{i\in\mathcal{I}}, \{\tilde{e}_i\}_{i\in\mathcal{I}})$ is a pair of dual frames for $\mathcal{H}$ if $\{e_i\}_{i\in\mathcal{I}}$ and $\{\tilde{e}_i\}_{i\in\mathcal{I}}$ are Bessel sequences satisfying (1.3). We denote by $\mathbb{Z}$ and $\mathbb{N}$ the set of integers and the set of positive integers respectively. Let $d \in \mathbb{N}$. We denote by $\mathbb{T}^d = [0, 1)^d$ the d-dimensional torus, and, for a Lebesgue measurable set E in $\mathbb{R}^d$, by $|E|$ its Lebesgue measure and χ_E the characteristic function of E respectively, and denote by τ the mapping from $\mathbb{R}^d$ to $\mathbb{T}^d$ defined by

$$\tau(x) = x - k \text{ for } x \in \mathbb{T}^d + k \text{ with } k \in \mathbb{Z}^d. \tag{1.4}$$

We denote by A^* its conjugate transpose for a $d\times d$ matrix A. A $d\times d$ matrix A is called an *expansive matrix* if it is an integer matrix with all its eigenvalues being greater than 1 in modulus. The *Fourier transform* of a function $f \in L^1(\mathbb{R}^d) \cap L^2(\mathbb{R}^d)$ is defined by

$$\hat{f}(\cdot) = \int_{\mathbb{R}^d} f(x)e^{-2\pi i\langle x,\, \cdot\rangle}dx$$

and extended to $L^2(\mathbb{R}^d)$ as usual, where $\langle\cdot, \cdot\rangle$ denotes the Euclidean inner product in $\mathbb{R}^d$. Similarly, the *inverse Fourier transform* of a function $f \in L^1(\mathbb{R}^d) \cap L^2(\mathbb{R}^d)$ is defined by

$$\check{f}(\cdot) = \int_{\mathbb{R}^d} f(x)e^{2\pi i\langle x,\, \cdot\rangle}dx$$

and extended to $L^2(\mathbb{R}^d)$ as usual. We define the *bracket product* $[f, g]$ of f, $g \in L^2(\mathbb{R}^d)$ by

$$[f, g](\cdot) = \sum_{k\in\mathbb{Z}^d} f(\cdot + k)\overline{g(\cdot + k)}$$

and the spectrum $\sigma(f)$ by

$$\sigma(f) = \left\{\xi \in \mathbb{T}^d : [\hat{f}, \hat{f}](\xi) > 0\right\}$$

Please cite this article in press as: Y.-Z. Li, J.-P. Zhang, Extension principles for affine dual frames in reducing subspaces, Appl. Comput. Harmon. Anal. (2017), https://doi.org/10.1016/j.acha.2017.11.006