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Flow equivalence and orbit equivalence for shifts of finite type and isomorphism of their groupoids

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ABSTRACT

We give conditions for when continuous orbit equivalence of one-sided shift spaces implies flow equivalence of the associated two-sided shift spaces. Using groupoid techniques, we prove that this is always the case for shifts of finite type. This generalises a result of Matsumoto and Matui from the irreducible to the general case. We also prove that a pair of one-sided shift spaces of finite type are continuously orbit equivalent if and only if their groupoids are isomorphic, and that the corresponding two-sided shifts are flow equivalent if and only if the groupoids are stably isomorphic. As applications we show that two finite directed graphs with no sinks and no sources are move equivalent if and only if the corresponding graph C^*-algebras are stably isomorphic by a diagonal-preserving isomorphism (if and only if the corresponding Leavitt path algebras are stably isomorphic by a diagonal-preserving isomorphism), and that two topological Markov chains are flow equivalent if and only if there is a diagonal-preserving isomorphism between the stabilisations of the corresponding Cuntz–Krieger algebras (the latter generalises a result of Matsumoto and Matui about irreducible topological Markov chains with no isolated points to a result about general topological Markov chains). We also show that for general shift spaces, strongly continuous orbit equivalence implies two-sided conjugacy.

1. Introduction

In their beautiful recent paper [20], Matsumoto and Matui proved that a simple Cuntz–Krieger algebra remembers the flow equivalence class of the irreducible shift of finite type defining it, provided that the canonical diagonal subalgebra is considered as a part of the data. A key tool for obtaining this groundbreaking result was the realisation that diagonal-preserving isomorphism translates directly to isomorphism

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of the groupoids associated to the shift spaces, reducing the problem to establishing that when two one-sided irreducible shifts of finite type are continuously orbit equivalent in the sense developed by Matsumoto, then the corresponding two-sided shift spaces are flow equivalent.

Having such rigidity results for C^*-algebras associated to general shift spaces of finite type would provide a better understanding of the classification problem for general Cuntz–Krieger algebras recently solved in [14] and [15]. From the point of view of symbolic dynamics, it is also of interest to determine the class of shift spaces for which continuous orbit equivalence implies flow equivalence.

The groupoid component of the proof in [20] has in [6] and [9] been generalised to a much more general setting, but the argument leading from diagonal-preserving isomorphism to flow equivalence in [20] goes through a deep result about the ordered cohomology of irreducible shifts of finite type by Boyle and Handelman ([4]) which does not readily extend to the reducible case. In addition, several of the arguments used in [20] rely on the assumption that the shifts of finite type in question do not contain isolated periodic points.

In the present paper we give a direct proof that continuously orbit equivalent shifts of finite type are also flow equivalent (Theorem 4.1) and thereby generalising [20, Theorem 3.5] from irreducible one-sided Markov shifts to general (possible reducible) shifts of finite type. We do that by producing a concrete flow equivalence from a given orbit equivalence between general shift spaces with continuous cocycles under added hypotheses on the given orbit equivalence and cocycles (Proposition 3.2), and then proving by methods related to the original proof in [20] that when the shift spaces are of finite type, then these hypotheses may always be arranged (Proposition 4.5 and Proposition 4.8).

As a corollary to Proposition 3.2, we generalise in Corollary 3.12 [19, Theorem 5.5] from irreducible topological Markov chains with no isolated points to general shift spaces by showing that for general shift spaces, strongly continuous orbit equivalence implies two-sided conjugacy.

We also prove that the groupoids of two one-sided shifts of finite type are isomorphic if and only if the shift spaces are continuously orbit equivalent (Theorem 5.1), and by combining this with a result of Matui [22] and results in [9] and [14], we obtain that these groupoids are stably isomorphic if and only if the corresponding two-sided shift spaces are flow equivalent (Theorem 5.3).

As applications, we show in Corollary 6.1 that the one-sided edge shifts of two finite directed graphs with no sinks and no sources are continuous orbit equivalent if and only if the corresponding graph C^*-algebras are isomorphic by a diagonal-preserving isomorphism (if and only if the corresponding Leavitt path algebras are isomorphic by a diagonal-preserving isomorphism), and we show in Corollary 6.3 that the graphs are move equivalent, as defined in [26], if and only if the corresponding graph C^*-algebras are stably isomorphic by a diagonal-preserving isomorphism (if and only if the corresponding Leavitt path algebras are stably isomorphic by a diagonal-preserving isomorphism).

We also apply our results to Cuntz–Krieger algebras and topological Markov chains and directed graphs of $\{0,1\}$-matrices and thereby generalise [20, Theorem 2.3] and [20, Corollary 3.8] from the irreducible to the general case (Corollary 7.1 and Corollary 7.2).

2. Definitions and notation

In this section we briefly recall the definitions of *shift spaces*, *shifts of finite type*, *continuous orbit equivalence of shift spaces*, and *flow equivalence of shift spaces*, and introduce notation.

We let $\mathbb{N}$ denote the set of positive integers, and $\mathbb{N}_0$ the set of non-negative integers.

2.1. One-sided shift spaces

A *one-sided shift space* (or *one-sided subshift*) is a closed, and hence compact, subset X of $\boldsymbol{a}^{\mathbb{N}_0}$, where $\boldsymbol{a}$ is a finite set equipped with the discrete topology and $\boldsymbol{a}^{\mathbb{N}_0}$ is equipped with the product topology, such that X is invariant by the shift transformation