Accepted Manuscript

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Alex Iosevich, Chun-Kit Lai, Azita Mayeli

PII: S1063-5203(17)30124-0
DOI: https://doi.org/10.1016/j.acha.2017.10.005
Reference: YACHA 1232

To appear in: *Applied and Computational Harmonic Analysis*

Received date: 30 November 2016
Accepted date: 24 October 2017

Please cite this article in press as: A. Iosevich et al., Tight wavelet frame sets in finite vector spaces, *Appl. Comput. Harmon. Anal.* (2017), https://doi.org/10.1016/j.acha.2017.10.005

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ACCEPTED MANUSCRIPT

Tight Wavelet Frame Sets in Finite Vector Spaces

Alex Iosevich, Chun-Kit Lai, Azita Mayeli

November 23, 2017

Abstract

Let $q \geq 2$ be an integer, and $\mathbb{F}_q^d$, $d \geq 1$, be the vector space over the cyclic space $\mathbb{F}_q$. The purpose of this paper is two-fold. First, we obtain sufficient conditions on $E \subset \mathbb{F}_q^d$ such that the inverse Fourier transform of 1_E generates a tight wavelet frame in $L^2(\mathbb{F}_q^d)$. We call these sets (tight) wavelet frame sets. The conditions are given in terms of multiplicative and translational tilings, which is analogous with Theorem 1.1 ([20]) by Wang in the setting of finite fields. In the second part of the paper, we exhibit a constructive method for obtaining tight wavelet frame sets in $\mathbb{F}_q^d$, $d \geq 2$, q an odd prime and $q \equiv 3 \pmod 4$.

Key words and phrases. Prime fields; finite vector space; wavelet frames; wavelet frame sets; tight frames; translational tiling; multiplicative tiling; rotational tiling; spectral set; spectrum; spectral pair

1 Introduction

A countable subset $\{x_k\}_{k\in I}$ of a Hilbert space $\mathcal{H}$ is said to be a *frame* if there exists two positive constants $A \leq B$ such that for any $x \in \mathcal{H}$

$$A\|x\|_{\mathcal{H}}^2 \leq \sum_{k\in I} |\langle x, x_k\rangle_{\mathcal{H}}|^2 \leq B\|x\|_{\mathcal{H}}^2.$$

The positive constants A and B are called lower and upper frame bounds, respectively. The frame is called a *tight frame* when we can take $A = B$ and it is called a *Parseval frame* if $A = B = 1$. One of most significant features of the frames that makes them practical and useful is their redundancy which has an important role, for example, in robustness. The frames also allow a localized representation of elements in the Hilbert space and they have been used for a number of years by engineers and applied mathematicians for purposes of signal processing and data compression. The notion of frames was first introduced by Duffin and Schaeffer [6]. Amongst the frames, tight frames play a fundamental role in the applications of frames due to their numerical stability. In this paper we aim to construct tight frames on the finite vector spaces over the finite fields which arise from dilation and translation of a function whose Fourier transform is characteristic function of a non-empty set.

In the classical setting, a function $\psi \in L^2(\mathbb{R}^d)$ is said to generate a *orthonormal wavelet basis* (resp. *wavelet frame*) if there is a set of $d \times d$ matrices $\mathcal{D} \subset GL(d, \mathbb{R})$ and a subset $T \subset \mathbb{R}^d$ such

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