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Infinite propagation speed of a weakly dissipative modified two-component Dullin-Gottwald-Holm system☆

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Abstract

We consider the infinite propagation speed of a weakly dissipative modified two-component Dullin-Gottwald-Holm (mDGH2) system. The infinite propagation speed is derived for the corresponding solution with compactly supported initial data does not have compact support any longer in its lifespan.

Keywords: Two-component peakon system, Weakly dissipative, Infinite propagation speed.
2010 MSC: 35Q53, 35G25, 35L05

1. Introduction

The modified two-component Dullin-Gottwald-Holm (mDGH2) system reads

$$\begin{cases} u_t - \alpha^2 u_{xxt} + c_0 u_x + 3uu_x + \beta u_{xxx} = \alpha^2(2u_x u_{xx} + uu_{xxx}) + \gamma\rho\bar{\rho}_x, & t > 0, \quad x \in \mathbb{R}, \\ \rho_t + (\rho u)_x = 0, \quad t > 0, \quad x \in \mathbb{R}, \end{cases} \tag{1.1}$$

where the constants $\alpha^2 (\alpha > 0)$ and β/c_0 are squares of length scales, $c_0 = \sqrt{h\gamma} \geq 0$ is the linear wave speed for undisturbed water resting at spatial infinity, here $h > 0$ denotes the mean fluid depth, $\gamma > 0$ denotes the downward constant acceleration of gravity in applications to shallow water waves, and $u(t, x)$ stands for the fluid velocity. The system (1.1) is expressed in terms of an averaged or filtered density ρ in analogy to the relation between momentum and velocity $\rho = (1 - \partial_x^2)(\bar{\rho} - \bar{\rho}_0)$, here $\bar{\rho}_0$ is a constant [10, 13, 15, 16].

The mDGH2 system (1.1) is a modified version of the following DGH2 system

$$\begin{cases} u_t - u_{xxt} + c_0 u_x + 3uu_x + \beta u_{xxx} - 2u_x u_{xx} - uu_{xxx} + \rho\rho_x = 0, & t > 0, \quad x \in \mathbb{R}, \\ \rho_t + (\rho u)_x = 0, \quad t > 0, \quad x \in \mathbb{R}, \end{cases} \tag{1.2}$$

which is derived in [7] by employing Ivanov's approach [9]. There are many work to study some qualitative analysis of the DGH2 system (1.2) [15, 16].

It is difficult to avoid energy dissipation mechanisms in experiments for real waves. For this reason Ott and Sudan [12] investigated how KdV equation has to be modified to include the effect of dissipation and the influence of dissipation to the solitary solution of KdV equation.

In this paper, we would like to consider the following weakly dissipative mDGH2 system

$$\begin{cases} u_t - \alpha^2 u_{xxt} + c_0 u_x + 3uu_x + \beta u_{xxx} + \lambda(u - \alpha^2 u_{xx}) = \alpha^2(2u_x u_{xx} + uu_{xxx}) + \gamma\rho\bar{\rho}_x, & t > 0, \quad x \in \mathbb{R}, \\ \rho_t + (\rho u)_x + \lambda\rho = 0, \quad t > 0, \quad x \in \mathbb{R}, \end{cases} \tag{1.3}$$

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