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Qualitative analysis for a diffusive predator–prey model with hunting cooperative

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HIGHLIGHTS

- A diffusive predator–prey model with hunting cooperative is proposed.
- Existence of interior equilibrium point is studied.
- Local stability of interior equilibrium point for non-spatial is discussed.
- Hopf bifurcation at positive constant steady state solution is studied.
- Turing instability of positive constant steady state solution is discussed.

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ABSTRACT

This paper investigates a predator–prey model that adds a cooperation term to the attack rate of the predator population. For the non-spatial model, the existence and stability of non-negative equilibrium points, and bifurcations are studied by choosing cooperation coefficient as control parameter. Our analytical results show that hunting cooperation can be beneficial to the predator population. However, as the cooperation coefficient increases, hunting cooperation can also destabilize the model and promote a sudden collapse of the predator population. For the spatial model, the stability of positive constant steady state solution, Hopf bifurcation and Turing instability are discussed. It is obtained that when the predation diffusion is not smaller than the prey diffusion, the spatial model can reserve the stable stability of the positive constant steady state solution. It is noted that the model without hunting cooperative does not generate Turing instability, while the model with hunting cooperative may generate Turing instability.

1. Introduction

Predator–prey interactions have been the hotspots in the research of mathematical biology so far [1–7]. In order to study the interaction between predator and prey, it is important to determine the specific form of the functional response that describes the amount of prey consumed per predator per unit of time it affects the population dramatically [8]. In the natural world, cooperative behavior within a population is a widespread and important phenomenon in biological systems [9]. It is noted that this behavior generally induce a positive relationship between the per capita growth rate and population density, that is, so-called Allee effect [10]. One of cooperative behaviors in predator is foraging facilitation among predators [11,12]. Many living organisms, such as lions [13,14], wolves [15] and African wild dogs [16], cooperate for hunting.

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As pointed out in [11], independence of a predator functional response of predator density, which means that any single predator affects the prey population growth rate independently of its conspecifics, that effects of two or more predators sum up, and that competition among predators for food occurs only through prey depletion, is hardly always true. An alternative possibility for the predator functional response to depend on predator density, largely neglected in current predator–prey theory, is the process inverse to predator interference, that is, foraging facilitation. It is clear that in the case of hunting cooperation, the functional response depends on both prey and predator densities. [12] argued that cooperative predators benefit from their behavior, so that the success of attacks on prey increases with predator density, and extended the classical Lotka–Volterra model by including hunting cooperation as follows

$$\begin{cases} \dfrac{\mathrm{d}N}{\mathrm{d}\tau} = rN\left(1-\dfrac{N}{K}\right) - N(\beta+\alpha P)P, \\ \dfrac{\mathrm{d}P}{\mathrm{d}\tau} = eN(\beta+\alpha P)P - mP, \end{cases} \tag{1}$$

where N and P are prey and predator densities respectively, K is the carrying capacity of prey, r is the intrinsic growth rate of prey, e is the conversion efficiency, m is the mortality rate of the predator, β is the rate of predation without hunting cooperative, and α is the cooperative coefficient. It is easy to see that when $\alpha = 0$, hunting cooperation disappears. K, r, e, m, β are positive constants, and α is a non-negative constant. The predator in model (1) is a specialist that feeds on one prey population, so that the growth rate of the predator is constrained by the prey population. By choosing appropriate scales for the variables of model (1), the number of parameters can be lessened. Considering dimensionless variables with the following scaling

$$u = \frac{e\beta}{m}N,\ v = \frac{\beta}{m}P,\ t = m\tau,$$

and model (1) is rewritten as follows

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = \sigma u\left(1-\dfrac{u}{\kappa}\right) - u(1+av)v, \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = u(1+av)v - v \end{cases} \tag{2}$$

with

$$\sigma = \frac{r}{m},\ \kappa = \frac{e\beta K}{m},\ a = \frac{\alpha m}{\beta^2}.$$

Spatial–temporal patterns appear almost everywhere in nature and their description and understanding still raise important and basic questions. The diffusion has been observed as causes of the spontaneous emergences of ordered structures, called patterns, in a variety of nonequilibrium situations [17]. Patterns generated in homogeneous environments are particularly interesting because they emerge from interactions in spatial scales that are much larger than the characteristic scale of individuals [18]. By incorporating diffusion terms, model (2) becomes

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1\triangle u + \sigma u\left(1-\dfrac{u}{\kappa}\right) - u(1+av)v,\ (x,t)\in\Omega\times(0,+\infty), \\ \dfrac{\partial v}{\partial t} = d_2\triangle v + u(1+av)v - v,\ (x,t)\in\Omega\times(0,+\infty), \\ \dfrac{\partial u}{\partial \mathbf{n}} = \dfrac{\partial v}{\partial \mathbf{n}} = 0,\ (x,t)\in\partial\Omega\times(0,+\infty), \\ u(x,0)=u_0(x)\ge 0,\ v(x,0)=v_0(x)\ge 0,\ x\in\Omega, \end{cases} \tag{3}$$

where d_1 and d_2 are the diffusion coefficients of prey and predator respectively, Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, $\mathbf{n}$ is the outward unit normal on $\partial\Omega$, and zero-flux boundary conditions imply that no external input is imposed from outside [19].

The non-spatial models with hunting cooperative have gained a lot of attentions [11,12,20,21]. For example, [12] has given the stability and bifurcation analysis of model (1) by numerical simulations. To the best of my knowledge, there is very little literature about the spatial model with hunting cooperative. This paper is devoted to studying non-spatial model (2) and spatial model (3) by choosing a as the control parameter, and determine the threshold values of a responding to the number of positive equilibrium points, stability and bifurcation respectively.

The rest of the paper is organized as follows. In Section 2, we discuss the existence and stability of equilibrium points for non-spatial model (2). In Section 3, the local stability and bifurcation for spatial model (3) at positive constant steady state solution are analyzed. In Section 4, some numerical simulations are given including bi-stability, periodic orbit. Finally, in Section 5 we draw the conclusion.