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Pinning outer synchronization of partially coupled dynamical networks with complex inner coupling matrices

Xuechen Li [a], Nan Wang [a], Jianquan Lu [b,*], Fuad E. Alsaadi [c]

[a] *School of Mathematics and Statistics, Xuchang University, Xuchang 461000, China*
[b] *School of Mathematics, Southeast University, Nanjing 210096, China*
[c] *Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia*

HIGHLIGHTS

- We investigate outer synchronization of partially coupled dynamical networks.
- The inner coupling matrices between coupled nodes can be non-full rank and also asymmetric.
- Our synchronization criteria can be theoretically proved to be less conservative.

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ABSTRACT

This paper investigates outer synchronization of partially coupled dynamical networks with complex inner coupling matrices via pinning impulsive controller. At first, we establish more realistic drive–response partially coupled networks, where the inner coupling matrices between coupled nodes can be non-full rank and also asymmetric. Then, a weighted-norm-based method is given to select the nodes that should be pinning controlled, and our synchronization criteria derived by this method can be proved to be less conservative. By using the regrouping method and our new average impulsive interval method, some efficient and less conservative synchronization criteria are derived. Our results show that the outer synchronization can be achieved by impulsively controlling a crucial fraction of nodes in the response network. Finally, numerical examples are exploited to illustrate the effectiveness of our theoretical results.

1. Introduction

In recent years, complex networks have attracted increasing attentions due to their wide applications in machine learning, information transmission, optimization, and so on. Complex dynamical networks consist of a large set of coupled nodes, in which each node represents an individual element and the edges represent the relations between the nodes [1–3]. Many research on complex dynamical networks have been addressed, such as synchronization, stability, and Hopf bifurcation analysis. Synchronization, as a typical collective behavior in networks, has received considerable attention in last two decades, since it is not only a common phenomenon occurring in many natural systems, but also has many applications in engineering. Synchronization means that all subsystems in coupled systems with different initial values give rise to a common behavior. Till now, many results about synchronization of coupled systems or complex dynamical networks have been addressed in various angles [4–16].

* Corresponding author.
E-mail address: jqluma@seu.edu.cn (J. Lu).

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Most of the research on synchronization of networks aforementioned focused on the inner synchronization, which is related to the collective behavior among all nodes within single network. Actually, there exist many other kinds of network synchronization, such as projective synchronization [17], cluster synchronization [18]. Different from the inner synchronization, there exists another kind of synchronization, regardless of happening of the inner synchronization, the corresponding nodes of two or more coupled networks could realize synchronization. This synchronization has been regarded as "outer synchronization". Outer synchronization is also ubiquitous in our daily life, such as, the spread of infectious diseases. Outer synchronization problems have attracted significant attention [19–21] since they were firstly studied by Li et al. [22]. Tang et al. [19] investigated two complex networks obtained outer synchronization by designing an effective adaptive controller. Lei et al. [21] studied the generalized matrix projective outer synchronization of non-dissipatively coupled time-varying complex dynamical networks. Wu et al. [20] investigated the generalized outer synchronization between two completely different networks under a nonlinear control scheme.

In complex dynamical networks, the initial states of the nodes are different. Thus nodes of the networks need to exchange their information with their neighbors to achieve synchronization. Note that most of the literature on network dynamical behaviors need the assumption that full state information of the nodes can be transmitted via the connections. Namely, there is no communication constraint. In fact, communication constraint exists widely due to environmental and physical limitation. As each node of the real network has multiple levels of information, the connections among any pairs of nodes should have multiple channels to transmit the corresponding state information. However, in many cases, only part of the channels can successfully transmit signals normally [23,24]. For example, in wireless networks, not all channel state information of users are well known to the transmitter. Hence, it is necessary and desirable to model and study partially coupled networks. To overcome the difficulties from communication constraint, regrouping method has been developed [23,24]. In [23], a class of stochastic dynamical networks with partial states, transmissions has been firstly established by introducing the concept of channel matrices, and some efficient synchronization criteria have been derived. In [25], Lu et al. investigated the outer synchronization of drive–response partially coupled networks. In [24,25], the channel matrix $R_{ij} = \mathrm{diag}\{r_{ij}^1, r_{ij}^2, \ldots, r_{ij}^n\}$ was defined as follows: if the sth channel of the connection from node j to node i is active, then $r_{ij}^s = 1$; otherwise, $r_{ij}^s = 0$. However, under many circumstances, this simplification is not in accord with the peculiarities of real networks. It means that there may exist communication from the lth component of node j to the kth component of node i. Thus, it is necessary to model this kind of more general networks and study the outer synchronization of the drive–response partially coupled networks. In this paper, we investigate outer synchronization of partially coupled dynamical networks with more complex inner coupling matrices, where the matrices can be non-diagonal and also asymmetric. So our network models will be more general, extensive and consistent with the real world than that of [25].

In order to achieve synchronization while the network itself cannot realize synchronization, many kinds of techniques have been developed in recent years. An effective method is adding controllers to the nodes in the response network, such as impulsive control. The stability of impulsive systems has been extensively investigated [26,27]. Impulsive control is an effective method for systems that cannot endure continuous control inputs. Due to its good properties, such as simplicity and flexibility, impulsive control scheme has been successfully applied in many disciplines, including neural networks, and population-growth models. When the network own a large set of high dimensional nodes, controlling all nodes becomes quite difficult to implement. Motivated by this practical consideration, only a small fraction of nodes is directly controlled has been proposed [28], which is called pinning control scheme [29–31]. This control scheme has been widely applied in complex networks [32,28,33–35]. Combining impulsive control and pinning control together, pinning impulsive control scheme has been introduced [36–40]. Pinning impulsive control is a powerful technique because it reduces the control cost to a certain extent. Therefore, it is necessary to study the synchronization problem under pinning impulsive control strategy. Lu et al. presented an analytical study of outer synchronization of partially coupled dynamical networks via pinning impulsive controller [25]. In [25], pinning impulsive controllers are chosen to be acted according to the norm of the error states, while in this paper, a weighted-norm-based method is given to select the nodes that should be pinning controlled. It can be proved that our weighted-norm-based method can make our synchronization criteria less conservative.

Using lower bound or upper bound of the impulsive intervals to characterize the frequency of impulses would lead to very conservative results. Hence, Lu et al. [41] has sought out a description about impulses' occurrence with the novel concept named average impulsive interval. Motivated by [41], furthermore, we introduced a new average impulsive interval in the form of limit [42], which can be used to characterize much wider range of impulsive sequences. In this paper, we will use our novel method named average impulsive interval to study outer synchronization of partially coupled dynamical networks via pinning impulsive controllers. Compared with existing results about synchronization of networks, the main contributions of this paper are listed as follows: (1) the channel matrix can be non-full rank, non-diagonal and asymmetric, and this makes our model more general. (2) a weighted-norm-based method is given to select the nodes that should be pinning controlled, and based on this method our results can be theoretically proved to be less conservative. (3) the impulsive sequence is characterized by our new concept of average impulsive interval, and this concept can be used to characterize much wider range of impulsive sequences.

The remainder of this paper is arranged as follows: In Section 2, we propose the problem of outer synchronization for two partially coupled dynamical networks and give some necessary preliminaries. In Section 3, several efficient outer synchronization criteria are established for partially coupled dynamical networks. In Section 4, numerical examples are given to illustrate our theoretical results. Finally, Section 5 presents the conclusion.

Notations: The standard notations will be used throughout this paper. The notation $X > (\geq, <, \leq)\ 0$ is used to denote a real symmetric positive-definite (respectively, positive-semidefinite, negative, and negative-semidefinite) matrix. The