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# Validation and comparison of circular coils' inductive coupling models

Guido De Angelis [a,*], A. Moschitta [b], A. De Angelis [b], P. Carbone [b]

[a] Regione Umbria, via M. Angeloni 61, 06124 Perugia, Italy
[b] Engineering Department, University of Perugia, via G. Duranti 93, 06125 Perugia, Italy

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In this paper, modeling of inductive coupling between circular coils is analyzed. We present a system for measuring the mutual inductance and the magnetic flux between resonant coils. We investigate the accuracy and precision against data obtained through the FastHenry2 simulation software and through experiments, when these are available (Pasku et al., 2017). The theoretical background is discussed, theoretical models are presented, and their properties are compared to those of other published models. A practical implementation is illustrated and employed to validate the considered numerical models. Obtained results demonstrate that the proposed modeling and experimental setup provide high-accuracy measurements of the induced voltage. Field measurements validate the considered numerical models with a discrepancy of less than 10% with respect to measurement results.



## 1. Introduction

The measurement and modeling of inductive coupling is gaining increasing interest in many research areas, which include Wireless Power Transfer [2–4], Electric Vehicles [5], Biomedical systems [6,7], Sensor Networks [8,9], and Magnetic Positioning Systems [10,1,11,12]. Various techniques were proposed to measure quantities of interest related to inductive coupling, such as the reflection coefficient between the transmitted and the received currents in coupled coils[13] and measurement of efficiency in power transfer systems [4,7]. Furthermore, Eddy-current non-destructive inspection is used to detect structural defects such as cracks and hidden corrosion in critical metal components, by measuring the mutual inductance of two coils [14]. In this context, the analysis of the mutual inductance between coupled inductors is of fundamental importance. Studies on mutual inductance involve the Rogowski coil [15], inductive position sensors [16], and impulse current measuring shunts [17].

In this paper, inductive coupling between two coils is considered. One of the coils, the field-generating coil, is assumed to be driven by a low-frequency sinusoidal excitation current. The induced voltage in the other coil, the sensor coil, is measured. The aim of this paper is to compare numerically and validate experimentally several models that are potentially applicable to model this interaction. We focus on two classes of models: the magnetic dipole moment model, [18,19], and the mutual inductance model [20,3] that is based on two methods. The former class is characterized by a lower computational complexity due to the nature of the employed approximations. Conversely, the latter class is more accurate but more computationally intensive. We compare two models in the latter class, based on two different approximations.

The magnetic dipole moment model is particularly suited for positioning systems, due to its low computational complexity that enables real-time parameter estimation. It was used to realize a system for tracking a receiving coil with an array of transmitting coils in [21]. Furthermore, the basic principle of AC magnetic 3-D ranging, using the magnetic dipole model with tri-axial coil arrangements, was described in [18]. To improve accuracy, in magnetic positioning as well as in the other applications based on inductive coupling, more complex models can be applied, including those based on mutual inductance.

This paper provides an extensive comparison between three published modeling methods from an analytical, numerical and experimental point of view, to evaluate their performance trade-offs. The main contribution of this paper is in the comparison of such models. While being meaningful for, e.g. designing and realizing magnetic positioning systems, a comparison between these modelling methods was never carried out. In order to compare the three methods on the same ground, we provide a comparison of the induced voltage predicted by each method with the measured voltage, under several geometrical configurations.

We focus on circular coils, since they are commonly used in the literature for such applications as magnetic positioning systems [1]. The geometries usually employed in this case are those with

* Corresponding author.
*E-mail address:* ing.guidodeangelis@gmail.com (G. De Angelis).

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circular symmetry, due to their isotropic properties. Other shapes do not provide the same spatial symmetry property offered by purely circular coils [7,22]. To realize simple sensors, we assume the coil with circular shape because in the case of coils with Archimedean spiral shape or square shape the resonant frequency depends on various parameters relating to the coil geometry and material properties. Finally, the results are compared between the proposed methods against a Finite Element Method (FEM). The FEM is a mathematical model based on the gridded surfaces of the 3-D coil. We applied this model using the FastHenry2 software [23,24] that is often considered as a reference in the literature [25,26].

The paper is structured as follows. In Section 2, the theoretical background and the development of the approximated measurement model are presented.

## 2. Theoretical background

To provide a theoretical foundation, in the following subsections the equations describing three different methods for modeling inductive coupling are provided: the first method is based on the magnetic dipole [18], the second method is based on an approximated mutual inductance formulation [27], and finally the third method represents a more refined mutual inductance formulation, which is derived in [20,3]. All models ultimately calculate the voltage induced in a receiving coil by a time-varying magnetic field generated by another coil. The numerical values of the voltage obtained by using the presented methods are compared with FEM.

### 2.1. Method I

Method I is based on the concept of the magnetic dipole [28]. Specifically, in near-field conditions [29,30], the magnetic field observed at the position $(x_u, y_u, z_u)$, generated by a transmitting coil driven by a sinusoidal current and centered at $(x_t, y_t, z_t)$, can be approximated by the equivalent dipole field using a phasor representation as follows [21,18]:

$$\mathbf{B} = \frac{\mu_0}{4\pi}\left(\frac{3(\mathbf{m}\cdot\mathbf{r})\mathbf{r}}{r^5} - \frac{\mathbf{m}}{r^3}\right)e^{-j\omega t}, \tag{1}$$

where $\mu_0$ is the vacuum permeability, $\mathbf{m}$ is the magnetic moment of a circular coil, $\omega$ is the angular frequency of the sinusoidal excitation, $t$ is time, $r$ is the distance between the center of the transmitting coil and the observation point, and $\mathbf{r}$ is the vector from the center of the transmitting coil to the observation point, given by

$$\mathbf{r} = [x_u - x_t \ y_u - y_t \ z_u - z_t]^T = r\overrightarrow{r}. \tag{2}$$

Here, the symbol $\overrightarrow{r}$ indicates the versor of the vector $\mathbf{r}$.

The magnetic moment of the transmitting coil is given by

$$\mathbf{m} = m\overrightarrow{\mathbf{m}} \tag{3}$$

where $m = \pi NIR^2$, $N$ is the number of turns, $I$ is the magnitude of the driving current, $R$ is the coil radius, and $\overrightarrow{\mathbf{m}}$ represents the unit vector perpendicular to the coil's plane. Without loss of generality, we assume that the transmitting coil lies on a plane parallel to the $xy$ plane, i.e.

$$\overrightarrow{\mathbf{m}} = [0\ 0\ 1]^T. \tag{4}$$

By substituting $B_T = \frac{m\mu_0}{4\pi}$ and (2)–(4) into (1), we have

$$\mathbf{B} = \frac{B_T}{r^3}\left(3\left(\begin{bmatrix}0\\0\\1\end{bmatrix}^T\begin{bmatrix}\frac{x_u-x_t}{r}\\\frac{y_u-y_t}{r}\\\frac{z_u-z_t}{r}\end{bmatrix}\right)\begin{bmatrix}\frac{x_u-x_t}{r}\\\frac{y_u-y_t}{r}\\\frac{z_u-z_t}{r}\end{bmatrix} - \begin{bmatrix}0\\0\\1\end{bmatrix}\right)e^{-j\omega t}. \tag{5}$$

$$\mathbf{B} = B_T\left(\frac{3(z_u-z_t)}{r^4}\begin{bmatrix}\frac{x_u-x_t}{r}\\\frac{y_u-y_t}{r}\\\frac{z_u-z_t}{r}\end{bmatrix} - \begin{bmatrix}0\\0\\\frac{1}{r^3}\end{bmatrix}\right)e^{-j\omega t}. \tag{6}$$

Therefore, the three components of the magnetic field along the $x$, $y$ and $z$ axes are

$$\begin{aligned} B_x &= B_T\left[\frac{3(z_u-z_t)(x_u-x_t)}{r^5}\right]e^{-j\omega t} \\ B_y &= B_T\left[\frac{3(z_u-z_t)(y_u-y_t)}{r^5}\right]e^{-j\omega t} \\ B_z &= B_T\left[\frac{3(z_u-z_t)(z_u-z_t)}{r^5} - \frac{1}{r^3}\right]e^{-j\omega t}. \end{aligned} \tag{7}$$

By imposing, without loss of generality, that the transmitting coil is centered in the origin and that the receiving coil is centered in a point in the $xz$ plane, i.e. $(x_t, y_t, z_t) = (0,0,0)$ and $(x_u, y_u, z_u) = (x, 0, h)$, the magnetic field can be calculated by using (7) as follows:

$$\mathbf{B} = \begin{bmatrix}\frac{3B_T h x_u}{\left(x_u^2+h^2\right)^{5/2}}\\0\\B_T\left(\frac{3h^2}{\left(x_u^2+h^2\right)^{5/2}} - \frac{1}{\left(x_u^2+h^2\right)^{3/2}}\right)\end{bmatrix}e^{-j\omega t} = \mathbf{B}'e^{-j\omega t}.$$

By assuming uniform flux on the surface of the receiving coil, as in [18], the magnitude of the voltage induced on a receiving coil centered at the observation point can be approximated as:

$$V = 2\pi f_0 N_c S_c \mathbf{B}'\cdot\overrightarrow{n_c}, \tag{8}$$

where $S_c$ is the receiving coil's area, $N_c$ is the number of turns in the receiving coil, and $\overrightarrow{n_c}$ is the versor describing the receiving coil orientation.

The model defined by (8) is valid for arbitrary positions and orientations of the two coils. However, by considering a specific geometric configuration, it is possible to obtain results that are more easily validated by experiments. In particular, if the coils lie on parallel planes, then the position of the receiving coil where the induced voltage becomes zero is easily derived as $x_{u,zero} = \sqrt{2}h$. This is the position in which the versor of the receiving coil is orthogonal to the field, thus $\mathbf{B}\cdot\overrightarrow{n_c} = 0$, Furthermore, the position of the local maximum of the induced voltage is calculated by differentiating the expression of the received voltage (8) as follows:

$$\frac{\partial V}{\partial x_u} = 0, \tag{9}$$

that results is $x_{u,max} = 2h$. The latter derivations will be validated by the experimental results described in Section 5.

### 2.2. Method II

The goal of Method II is to approximate the mutual inductance between two coils, arbitrarily positioned in space as shown in Fig. 1. The approximation is obtained as a scaled version of the mutual inductance between two filaments. Assume, without loss of generality, that the transmitting filament is centered at the origin of the coordinate system and lies on the $xy$ plane. The receiving filament, of radius $R_s$, lies in an inclined plane $\lambda$ whose general equation is given by

$$\lambda \equiv ax + by + cz + d = 0. \tag{10}$$

The mutual inductance between inclined circular filaments arbitrarily positioned in space with respect to each other was derived in [27]: