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# A matheuristic based on Lagrangian relaxation for the multi-activity shift scheduling problem

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ABSTRACT

The multi-activity shift scheduling problem involves assigning a sequence of activities to a set of employees. In this paper, we consider the variant where the employees have different qualifications and each activity must be performed in a specified time window; i.e., we specify the earliest start period and the latest finish period. We propose a matheuristic in which Lagrangian relaxation is used to identify a subset of promising shifts, and a restricted set covering problem is solved to find a feasible solution. Each shift is represented by a context-free grammar. Computational tests are carried out on two sets of instances from the literature. For the first set, the matheuristic finds a solution with an optimality gap less than 0.01% for 70% of the instances and improves the best-known solution for 16% of them; for the second set, the matheuristic reaches the best-known solutions for 55% of the instances and finds better solutions for 37.5% of them.



## 1. Introduction

For many companies, designing a schedule for their employees is complex. An efficient schedule impacts directly on the quality of customer service, the human resource management, and the employee satisfaction. The constraints involved in the assignment of shifts to employees include employee availability, qualifications, and preferences; company policies; the number of activities; and collective agreements. Additionally, the assignment of the activities depends on the forecast demand over the planning horizon. A shortage of employees for a particular activity may result in customer losses, and an excess number of employees performing the same activity may lead to inactivity.

In this context, we design a matheuristic to efficiently solve the multi-activity shift scheduling problem (MASSP), also known in the literature as the personalized multi-activity shift scheduling problem, which involves assigning a sequence of activities to each employee. The goal is to minimize the total cost of undercovering or overcovering activities in each period of the planning horizon. Each activity has a different demand in each period; the same activity can be assigned to multiple employees and it must be performed in a specified time window. Other side constraints such as employee availability, preferences, and qualifications are taken into account.

To the best of our knowledge, few researchers have addressed this problem. Demassey, Pesant, and Rousseau (2006) solve a set covering problem (SCP) and use a column generation approach, where the subproblems are modeled using automatas and constraint programming. Lequy, Bouchard, Desaulniers, Soumis, and Tachefine (2012a) consider the case where the shifts are built a priori. They propose three integer programming models, a branch and bound method for small instances, and a rolling-horizon heuristic for large instances. Quimper and Rousseau (2010) use formal languages and context-free grammars to model the constraints of each shift; they design two operators within a large-neighborhood search. Dahmen and Rekik (2012) propose a hybrid method based on tabu search: a branch and bound algorithm is used in the improvement, intensification, and diversification phases of the procedure. Côté, Gendron, Quimper, and Rousseau (2011a), Côté, Gendron, and Rousseau (2011b, 2013) present two grammar-based models and a column generation embedded into a branch-and-price approach. Computational tests of Côté, Gendron, and Rousseau (2013) with the instances of Demassey et al. (2006) and Lequy et al. (2012a) show that their approach improves the solutions reported in the literature. The branch-and-price al-

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gorithm is able to solve instances with up to 100 employees and 10 activities over a planning horizon of 7 days, however in the worst case it takes more than two hours to obtain a solution with a relative gap of 1%. Finally, Restrepo, Lozano, and Medaglia (2012)) present a column generation algorithm coupled with an auxiliary shortest path problem with resource constraints for an application in Bogota, Colombia, where employees must be assigned to car parks.

Elahipanah, Desaulniers, and Lacasse-Guay (2013) present a two-phase heuristic for a variant of the MASSP in which multiple tasks are considered. The first phase solves a mixed integer linear model to generate partial shifts by assigning the tasks, and the second uses a rolling-horizon procedure to assign the activities. Lequy, Desaulniers, and Solomon (2012b) propose a two-stage heuristic for the multi-task variant. The first phase assigns the tasks using a mixed integer linear model, and the second assigns activities and reassigns the tasks using a column generation heuristic. Boyer, Gendrom, and Rousseau (2014) present an extension of the branch-and-price algorithm of Côté et al. (2013) for the MASSP with multiple tasks. The authors compare two formulations for the precedence constraints on the tasks and three branching strategies.

Lagrangian relaxation has been extensively used to solve the SCP; the derived reduced costs are often used to discard variables. Balas and Ho (1980) design a branch and bound for the SCP based on the Lagrangian relaxation of the problem. Balas and Carrera (1996) use the same approach to derive a heuristic: at each iteration of the branch and bound algorithm, variables are heuristically fixed to reduce the problem size. Ceria, Nobili, and Sassano (1998) and Caprara, Fischetti, and Toth (1999) also investigate different diving strategies built on the solution of the Lagrangian relaxation.

Umetani and Yagiura (2007) solve the SCP using a variant of column generation, called the shifting method (Bixby, Gregory, Lustig, Marsten, & Shanno, 1992). This approach uses a greedy algorithm to produce feasible solutions and the Lagrangian multipliers to fix variables. Caserta (2007) proposes a tabu search algorithm that uses Lagrangian relaxation in the intensification phase to fix variables.

We propose a matheuristic based on context-free grammars and Lagrangian relaxation, where the set of shifts explored by the subgradient method is integrated into the SCP to find a high-quality solution. We perform computational experiments on a large set of instances from Demassey et al. (2006) and Lequy et al. (2012a). The results are compared to the best-known solutions in the literature which, to the best of our knowledge, are due to Côté et al. (2013). Our approach also provides dual bounds that can be used to measure the quality of the solutions. Furthermore, the proposed procedure is able to find good quality solutions in a shorter processing time than the one reported by Côté et al. (2013).

The paper is organized as follows. We describe the MASSP in Section 2 and discuss the context-free grammar model used to represent the feasible shifts. Section 3 presents the solution method, and Section 4 shows the experimental results. Section 5 provides concluding remarks.

## 2. Multi-activity shift scheduling problem

The MASSP assigns shifts $s \in \Omega^e$ to employees $e \in E$, where $\Omega^e$ represents the set of feasible shifts for employee $e$. The objective is to cover at minimum cost the demand $b_{ia}$ for each activity $a \in A$ in period $i \in I$. We assume that the employees have differing availabilities and qualifications (these determine the set of activities that the employee is able to perform). The set of feasible shifts is determined by the characteristics of each employee and the company policies. Shift $s$ for employee $e$ is feasible if $e$ performs only activities for which he/she is qualified and $s$ satisfies the duration constraints and includes the required rest periods. Moreover, each activity in $s$ must be performed in a given time window.

The planning horizon $I$ is discretized into periods of equal length. Shift $s$ is represented by the sequence of activities that the employee performs; this sequence can include lunch periods or breaks. We associate a cost $c_s^e \geq 0$ with each $s \in \Omega^e$. This includes the cost of transitions between activities during a shift. Furthermore, we allow undercovering and overcovering of the activities, with associated penalties denoted by $c_{ia}^u$ and $c_{ia}^o$, respectively. Parameter $\delta_{ias}^e \in \{0, 1\}$ indicates whether or not activity $a$ is in shift $s$ in period $i$ for employee $e$.

Côté et al. (2013) propose the following SCP:

$$\textbf{(SCM)}\ \text{Min} \quad z = \sum_{e\in E}\sum_{s\in\Omega^e} c_s^e x_s^e + \sum_{i\in I}\sum_{a\in A}\left(c_{ia}^u u_{ia} + c_{ia}^o o_{ia}\right) \tag{1}$$

subject to

$$\sum_{e\in E}\sum_{s\in\Omega^e} \delta_{ias}^e x_s^e + u_{ia} - o_{ia} = b_{ia} \quad \forall i \in I, a \in A \tag{2}$$

$$\sum_{s\in\Omega^e} x_s^e = 1 \quad \forall e \in E \tag{3}$$

$$x_s^e \in \{0, 1\} \quad \forall e \in E, s \in \Omega^e \tag{4}$$

$$u_{ia} \geq 0 \quad \forall i \in I, a \in A \tag{5}$$

$$o_{ia} \geq 0 \quad \forall i \in I, a \in A \tag{6}$$

where:

- $x_s^e = 1$ if shift $s$ is assigned to employee $e$, and 0 otherwise.
- $u_{ia}$ represents the undercovering of activity $a$ in period $i$.
- $o_{ia}$ represents the overcovering of activity $a$ in period $i$.

The objective (1) minimizes the cost of assigning a shift to an employee and the costs of overcovering and undercovering. Constraints (2) ensure that the demand for each activity $a$ in each period $i$ is satisfied. Constraints (3) ensure that a shift is assigned to each employee.

To model the set of feasible shifts $\Omega^e$, we use a context-free grammar, as suggested by Côté et al. (2013). We summarize the model below; for further details, see (Hopcroft, Motwanu, & Ullman, 2001). Note that the context-free grammar is used to represent the set $\Omega^e$, and this set is not explicitly generated.

A context-free grammar $G$ is defined by the tuple ($\Sigma$, $N$, $P$, $S$) where

- $\Sigma$ is an alphabet of symbols, called terminals;
- $N$ is a set of nonterminal symbols;
- $P$ is a set of productions of the form $X \to \alpha$, where $X \in N$ and $\alpha$ is a sequence of terminal and nonterminal symbols;
- $S$ is the starting nonterminal symbol.

The context-free grammar generates a set of sequences called language, where these sequences are named words and are conformed only by terminal symbols. A production rule is a relationship between the nonterminals symbols and a sequence of terminal/nonterminal symbols. In this context, the set $P$ defines whether a word belongs to the language or not, by checking if it can be derived from $S$ using these production rules. The nonterminal symbols can be seen as transition symbols given that, depending on the productions, these symbols produce a subsequence of terminal and/or nonterminal symbols.

A shift can be represented as a sequence of terminal symbols giving the activities to be performed by the employee. The position of the symbol in the sequence corresponds to the period in