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Wavelet-based option pricing: An empirical study

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ABSTRACT

In this paper, we scrutinize the empirical performance of a wavelet-based option pricing model which leverages the powerful computational capability of wavelets in approximating risk-neutral moment-generating functions. We focus on the forecasting and hedging performance of the model in comparison with that of popular alternative models, including the stochastic volatility model with jumps, the practitioner Black–Scholes model and the neural network based model. Using daily index options written on the German DAX 30 index from January 2009 to December 2012, our results suggest that the wavelet-based model compares favorably with all other models except the neural network based one, especially for long-term options. Hence our novel wavelet-based option pricing model provides an excellent non-parametric alternative for valuing option prices.

1. Introduction

Since the seminal work of Black and Scholes (1973), huge progress has taken place in the theoretical and empirical option valuation literature that has greatly advanced our understanding of the options market as a place for trading information and gauging investor expectation. A large number of parametric and nonparametric methods have been developed to relax one or more restrictions of the original Black–Scholes model.

One avenue for extending the Black–Scholes model is to develop nonparametric models that are better at capturing the volatility smile and the literature has seen innovative methods in this direction. Wavelets are well-known for their remarkable ability in numerical approximation and the wavelet-based option pricing model developed in Ma (2011) leverages this. It approximates the implied risk-neutral moment-generating functions (MGF) thus offering a novel approach in the nonparametric option pricing literature. Unlike many other nonparametric option pricing models that require a large collection of data, the wavelet-based pricing model is computationally efficient and requires only a reasonable amount of different strikes. Using numerical experiments, Haven, Liu, Ma, and Shen (2009) demonstrate that this model is able to evaluate and forecast option prices with great precision.

In this paper, we contribute to the literature by taking this further to empirically compare the forecasting and hedging performance of the wavelet-based model with three other well-established models, namely the parametric stochastic volatility model with jumps (SVJ), the practitioner Black and Scholes model (PBS), and the hybrid neural network based model (NN), which is a combination of the neural network method and the Black–Scholes model.[2] We focus on the key research questions of whether the excellent performance of the wavelet-based model in simulation still remains in the crucial test using market data, and how its empirical performance compares with that of widely-accepted models in the literature.

To the best of our knowledge, this is the first study that subjects the wavelet-based option pricing model to market data. We use daily index options written on the DAX-30 index, a major financial index in Europe, from January 2, 2009, to December 28, 2012. Our main empirical findings can be summarized as follows. In the out-of-sample forecasting exercises ranging from one- to ten-day ahead, the wavelet-based model outperforms the SVJ for calls and exhibits similar performance for puts for all option maturities (short-, medium- and long-term) across all forecasting

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horizons. It also outperforms the PBS model in forecasting long-term options. This is the case regardless of whether the market is in a turbulent state with evident jumps in the underlying stock index. The NN model is always the best-performing model. In the hedging exercise, the performance of the wavelet-based model is second only to that of the NN and exhibits substantially smaller hedging error than the SVJ and PBS model.[3] Our strong empirical evidence substantiates the wavelet-based option pricing model as a credible alternative in the option valuation literature.

The rest of the paper is organized as follows. In Section 2, we review the relevant literature that motivates our study. Section 3 introduces the wavelet-based option pricing model, the SVJ model, the PBS model, and the neural network based model and how we conduct the hedging exercise. In Section 4, we describe data and analyze empirical results. Finally, Section 5 concludes.

2. Literature review

In this section, we review two strands of the literature to which our paper makes a contribution: the literature of option valuation and that of the wavelet method and its applications.

In the parametric option pricing literature, researchers have identified a number of priced factors essential in capturing the volatility smile, which has become a stylized fact since the market crash of 1987. For example, volatility is shown to relate negatively to the underlying asset returns and that delta-hedged portfolios of options and the underlying stocks produce statistically significant negative returns (see, for instance, Bakshi & Kapadia, 2003, Coval & Shumway, 2001, Heston, 1993, and Wong & Lo, 2009). Moreover, the random and unexpected jumps are also found to command significant risk premium in the options market (Bates, 1996; 2000; Cai & Kou, 2011; Pan, 2002). Another priced factor worth noting is the demand pressure in the market, which affects option prices in incomplete markets (Gârleanu, Pedersen, & Poteshman, 2009).

Bakshi, Cao, and Chen (1997) propose a closed-form parametric option pricing model that simultaneously admits the stochastic volatility risk, the jump risk, and the stochastic interest rate risk. One or more risks can be singled out by setting the parameters of the remaining risk factors to zero so that the importance of each risk factor can be closely investigated. Based on the pricing, forecasting, and hedging performance of nested models, they show that the stochastic volatility and jumps are of first-order importance when it comes to accommodating the volatility smile observed in the market.

Another prominent option pricing model is the practitioner Black–Scholes model (PBS). In PBS, a nonlinear deterministic volatility function (DVF) is employed to estimate volatility values and account for deviations from the assumptions of the original Black–Scholes model (Derman & Kani, 1994a, 1994b; Dumas, Fleming, & Whaley, 1998; Rubinstein, 1994). A theoretical justification of the PBS approach as a reduced-form approximation of an unknown structural model is provided by Berkowitz (2002). As discussed in Andreou, Charalambous, and Martzoukos (2014), the PBS model is widely used among practitioners as it is effective in mitigating the volatility smile anomaly and easy to implement (Berkowitz, 2010; Christoffersen & Jacobs, 2004; Christoffersen, S.L., & Jacobs, 2009).

Parallel to the intensive interest in the parametric option pricing literature, a large number of nonparametric models have also been developed. Although the nonparametric models lack the economic interpretation that the parameters contain in the parametric family of models, they are often more flexible as they impose no prior assumption on the underlying asset process. These include the flexible distribution method (Rubinstein, 1994), the cubic spline method (Shimko, 1994), which is further developed by Bliss and Panigirtzoglou (2002), the kernel estimation method (Aït-Sahalia & Duarte, 2003; Aït-Sahalia & Lo, 1998; Birke & Pliz, 2009), the neural network method (Andreou, Charalambous, & Martzoukos, 2008; Garcia & Gençay, 2000; Hutchinson, Lo, & Poggio, 1994), and the ϵ-arbitrage replicating portfolio method (Bandi & Bertsimas, 2014).

The neural network method is a popular method in option pricing. It is first introduced by Hutchinson et al. (1994) to estimate the unknown pricing formula for derivative securities and further employed for pricing derivatives in Lajbcygier, Boek, and Palaniswami (1996), Garcia and Gençay (2000), and Yao and Li (2000). They all show that when sufficiently trained, neural network method can adapt to changing market conditions and achieve remarkable accuracy. Andreou et al. (2008) combine the neural network method and the parametric option pricing models and show that the combined hybrid neural network model works even better than the pure neural network method. The neural network methods are highly data-intensive and require large amount of historical prices to obtain a reasonably well-trained network.

A more recent addition to this growing literature is Ma (2011). This book develops a nonparametric option pricing model that focuses on approximating the implied risk-neutral MGF of the underlying asset returns using wavelets. The risk-neutral MGF has a number of advantages compared with the implied risk-neutral PDF although there is a one-to-one relationship between them. For example, the MGF is more tractable when jumps are present in the underlying price process; the MGF obtained from options is a continuous function; the MGF can be used to obtain all the statistical moments of the underlying asset distributions and the preference parameter of the utility function; and out-of-sample options with different maturity dates can be directly estimated using the risk-neutral MGF.[4]

Ma (2011) represents another effort in applying the wavelet method, already a popular tool in science and engineering, in the area of economics and finance.[5] As pointed out in Haven et al. (2009) and Haven, Liu, and Shen (2012), there are mainly three types of application of wavelet methods in finance and economics. First of all, wavelets are used for multi-scaling analysis. For example, Ramsey and Lampart (1998a,b) use the wavelet method to analyze the relationship between economic variables at different scales and suggest that the relationship changes over different time horizons. Gençay, Selçuk, and Whitcher (2001a, 2001b, 2003, 2005) employ the wavelet multi-scaling approach to examine intra-day seasonalities, foreign exchange volatilities, and systematic risk. Weron (2009) implements the wavelet method to de-seasonalize electricity prices. More examples include Zapart (2002), Connor and Rossiter (2005), Kim and In (2005), Mitra (2006), In and Kim (2006), Fernandez (2006), Lien and Shrestha (2007), Gallegati and Gallegati (2007), and Nikkinen, pynnönen, Ranta, and Vähämaa (2011).

Secondly, wavelets are used to de-noise raw data. Capobianco (1999, 2001) show that wavelets as a pre-processing de-noising tool are useful for improving volatility analysis. The superior de-noising ability of wavelets is also recognized in Haven et al. (2012) which apply the wavelet method to de-noise option prices before estimating the implied risk-neutral PDF from the option

[3] We have conducted the same empirical analysis using options written on the FTSE 100 index in the UK and the Hang Seng index in Hong Kong over the same sample period. We obtain qualitatively similar results that we do not report to conserve space. These are available upon request from the authors.

[4] See Haven et al. (2009) for detailed properties of the risk-neutral MGF.

[5] For excellent reference for applications of the wavelet method in finance and economics, see Gençay, Selçuk, and Whitcher (2002). See also Percival and Walden (2000) for applications of the wavelet method in the time series analysis.