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Understanding the varying discharge rates of lognormal particle size distributions from a hopper using the Discrete Element Method

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ABSTRACT

The focus of this study is on understanding the varying discharge rates of lognormal particle size distributions (PSDs) with the same mean particle diameter. Discrete element method (DEM) is used to simulate lognormal PSDs with the same arithmetic mean of the particle diameter of 5 mm but varying PSD widths ($\sigma/\mu = 10\%$ to 70%) in a 3D conical hopper. Four highlights are noted: (i) the Beverloo correlation and others modified to account for various particle properties predict the discharge rates of lognormal PSDs poorly, which underscores the need for more understanding on the influence of PSD width; (ii) the velocity vectors are less uniformly downwards in the hopper for the wider PSDs, which results in the slowing down of the discharge rate; (iii) the radial particle velocity, and both the radial and vertical particle angular velocity increase with PSD width throughout the first half of the hopper discharge; and (iv) the collision force magnitudes are greater for the wider PSDs, and the cross-sectional fluctuations of the collision forces at the cone height increases with PSD width. The increase of the magnitudes of these particle characteristics (namely, radial particle velocity, angular velocity, and collision force) with PSD width underlies the decreasing hopper discharge rate.

1. Introduction

Hoppers are common apparatuses for the storage and discharge of particles in various industries (e.g., pharmaceutical, chemical, and food). Despite the seeming simplicity of function, flow problems including erratic or stunted flow, dead zones, segregation, and dust explosions are prevalent [1–3]. To resolve such issues, an in-depth understanding of the mechanisms underlying the different discharge behaviors [4,5], for example, due to different hopper configurations [6–8] and different particle properties [9–13], is necessary. To this end, both experimental and simulation efforts have been dedicated to investigating the discharge characteristics of hoppers [1,14–16]. The discrete element method (DEM) provides an effective method of tracking each particle numerically and has been widely accepted to study dense particulate behavior, which was initially proposed by Cundall and Strack [17]. Due to the dominance of collisions and significant shear in the hopper, particle-particle interactions have to be appropriately accounted for [18].

Previous DEM studies focusing on discharge behaviors in the hopper have provided some understanding on monodisperse [19,20], binary-size mixtures [21–23] and multi-size mixtures [24,25]. Li et al. [19] found good agreement between DEM simulation and experimental results on the particle trajectory for a high-temperature monodisperse system. Yu and Saxén [20] found that the velocity distribution is uniform except near the orifice and the static particle-wall friction is important for the flow pattern of the monodisperse flow. Rahman et al. [21] concluded that, for binary mixtures, the degree of segregation is higher in a three-dimensional system compared to a two-dimensional system and flowability is closely tied to segregation patterns. Combarros et al. [22] found that particle size has a more dominant effect than particle shape on segregation and that the critical DEM parameters in segregation simulation are static and rolling friction. Ketterhagen et al. [23] investigated the effects of the particle diameter ratio, density ratio, fines mass fraction, hopper wall angle, hopper cross-sectional shape, and the initial fill conditions of a binary-size mixture, and found that the DEM model can predict well the segregation during hopper discharge. Wu et al. [24] found that the flowability of the particles is tied to the hopper slope and the mass fraction of small particles. Yu and Saxén [25] concluded the filling method, diameter ratio of fine to coarse, wall-particle static and rolling friction, inter-particle rolling friction as well as mass fraction of fine particles are the most important factors affecting the extent of segregation during the discharging process. Anand et al. [26,27] further revealed that the coefficient of restitution and hopper width have negligible influence on the discharge rate, whereas friction, hopper angle, outlet width and particle size distribution (PSD) have more significant impact. Arteaga and Tüzün [28,29] concluded

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Table 1
The forces and torques acting between particle *i* and particle *j*.

Forces & torques	Equations
Normal contact force	$\boldsymbol{F}^n_{c,ij} = 4/3Y^*(r^*)^{1/2}{\delta_n}^{3/2}\boldsymbol{n}_{ij},$
Normal damping force	$\boldsymbol{F}^n_{d,ij} = 2\sqrt{5/6}\beta\sqrt{S_n m^*}\boldsymbol{v}^n_{ij},$
Tangential contact force	$\boldsymbol{F}^t_{c,\ ij}= 8G^*\sqrt{r^*\delta_n}\boldsymbol{t}_{ij},$
Tangential damping force	$\boldsymbol{F}^t_{d,ij} = 2\sqrt{5/6}\beta\sqrt{S_t m^*}\boldsymbol{v}^t_{ij},$
Torque by tangential forces	$\boldsymbol{M}_{t,\ ij} = R_{ij}\times(\boldsymbol{F}^t{}_{c,\ ij} + \boldsymbol{F}^t{}_{d,\ ij}),$
Rolling friction torque	$\boldsymbol{M}_{r,\ ij} = \mu_{r,\ ij}\lvert\boldsymbol{F}^n{}_{c,\ ij} + \boldsymbol{F}^n{}_{d,\ ij}\rvert\boldsymbol{\omega}_{t,\ ij}/\mid\boldsymbol{\omega}_{t,\ ij}\mid,$

where, $1/Y^* = (1-v_i^2)/Y_i + (1-v_j^2)/Y_j$, $1/r^* = 1/r_i + 1/r_j, \beta = \ln(e)/\sqrt{\ln^2(e)+\pi^2}$, $S_n = 2Y^*\sqrt{r^*\delta_n}$, $1/G^* = 2(2+v_i)(1-v_i)/Y_i-2(2+v_j)(1-v_j)/Y_j, S_t = 8G^*\sqrt{r^*\delta_n}$.
Note that *Y* is Young's modulus, *v* is Poisson's ratio, *G* is shear modulus and *r* is particle radius.

Table 2
Details of physical parameters in the simulation.

Parameter	Value
σ/μ (−)	10, 30, 50 and 70%
Number (−)	181,483, 171,259, 145,734, 102,202 and 67,406
Density, ρ_p (kg/m^3)	2460
Initial particle bed height (m)	0.474
Cone height (m)	0.052
Diameter of cylindrical section of hopper (m)	0.24
Diameter of outlet (m)	0.06
Angle of cone (°)	60
Young's modulus, *Y* (GPa)	0.1
Poisson ratio, *v* (−)	0.25
Coefficient of particle-wall static friction (−)	0.6
Coefficient of particle-wall rolling friction (−)	0.05
Coefficient of particle-wall restitution (−)	0.99
Coefficient of particle-particle static friction (−)	0.6
Coefficient of particle-particle rolling friction (−)	0.01
Coefficient of particle-particle restitution (−)	0.99
Time step, Δt (s)	2×10^{-5}

that the discharge rates were found to increase with increasing mass fraction of fines, with the extent of this increase being a strong function of the size ratio of the constituents. Our previous study [30] found the lognormal PSD width has a dominant influence on the discharge rate in the hopper, with the decrease in the discharge rate significant at approximately 36% when the PSD changes from a monodisperse to a lognormal PSD width (defined as ratio of standard deviation of the particle size to mean particle diameter) of 70%. The dependence of discharge rate and wall stress on particle interactions have been investigated using the discrete element model (DEM), and it was found that the proper accounting of inter-particle interactions is critical for more accurate predictions of discharge rates [7]. The effect of differential pressure (ΔP) on discharge rate in a conical hopper was experimentally studied, which indicated that increases in ΔP increased discharge rate up to a critical ΔP after which the discharge regime destabilized, and that the critical ΔP depended on hopper orifice diameter and physical properties of the particles like size distribution and particle density [5]. Recently, the discharge rate in conical hoppers was assessed using an elastoplastic model implemented with the Eulerian finite element method (FEM) approach, which demonstrated that material dilation and internal frictional angle markedly affect the discharge rate particularly for steep cones (cone angle <45°) [31]. Moreover, the behavior of super-quadric particles during hopper discharge was found to be such that the discharge rate decreases with increasing blockiness, particle friction or aspect ratio [32]. Collectively, these studies imply that particle properties like size and size distribution have a non-negligible impact on the hopper discharge rate.

The Beverloo correlation [33] represents one of the most popular for predicting hopper discharge rates, which was subsequently modified to account for varying particle properties. Because it was experimentally found that the correlation only worked well for coarser particles but not the finer ones (d ≤500 μm), the correlation was modified based on the different momentum and drag of the particles [34]. Another group investigated finer particles (50 μm≤ d ≤500 μm) and found that, although the developed correlation for discharge rates has the right relationship with particle size, it gives quantitative values about twice larger than the experimentally measured values [13]. The shape of the particles was also experimentally found to have a significant effect on the flow rate, and therefore the Beverloo correlation was further incorporated with the shape parameters [35]. A recent work [36] investigated the influence of orifice shape on the flow rate in flat-bottom hoppers numerically and experimentally, and found the data can be well-fitted with a modified Beverloo's formula. Also, with regards to a wider range of particle sizes, the Beverloo correlation was extended to binary-size mixtures [12] based on earlier experimental data [9] and also polydisperse mixtures by including the differential pressure gradients [14,37].

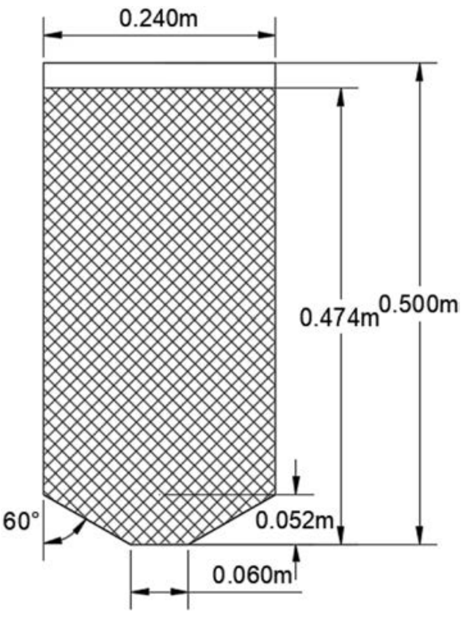

Fig. 1. Geometrical description of the hopper with an initial packed bed height of 0.474 m.

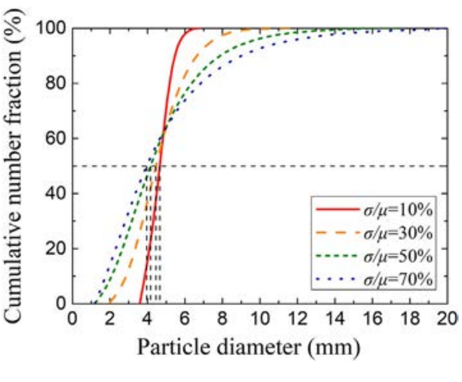

Fig. 2. Cumulative weight fraction of particles with mono-sized, lognormal distribution with $\sigma/\mu = 10\%$, 30%, 50%, and 70%. The black dashed lines represent the 50th percentile (or median) mass-based diameter.