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On the explicit upper and lower bounds for the number of zeros of the Selberg class

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Abstract

In this paper we prove explicit upper and lower bounds for the error term in the Riemann-von Mangoldt type formula for the number of zeros inside the critical strip. Furthermore, we also give examples of the bounds.

Keywords: Selberg class, Riemann-von Mangoldt type formula, Explicit upper and lower bounds

1. Introduction

The Selberg class S, defined by A. Selberg [17], consists of functions $\mathcal{L}(s) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s}$ which satisfy the following conditions:

1. Ramanujan hypothesis: For any $\epsilon > 0$ we have $|a(n)| \ll_\epsilon n^\epsilon$.
2. Analytic continuation: There is an integer $k \geq 0$ such that $(s-1)^k \mathcal{L}(s)$ is an entire function of finite order.
3. Functional equation: There exists a positive integer f and a real number Q and for integer $j \in [1, f]$ there are positive real numbers λ_j and complex numbers ω, μ_j, $d_{\mathcal{L}} = 2\sum_{j=1}^{f} \lambda_j$, $\lambda = \prod_{j=1}^{f} \lambda_j^{2\lambda_j}$ where $|\omega| = 1$ and $\Re(\mu_j) \geq 0$ which satisfy

$$\Lambda_{\mathcal{L}}(s) = \omega \overline{\Lambda_{\mathcal{L}}(1-\bar{s})}$$

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