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An Eichler–Zagier map for Jacobi cusp forms on $\mathcal{H} \times \mathbb{C}^{(g,1)}$

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A B S T R A C T

We define an Eichler–Zagier map $\mathcal{Z}_M$ on the space of Jacobi cusp forms of matrix index M and discuss its mapping properties. If the order of M is congruent to 1 mod 8 then we show that $\mathcal{Z}_M$ maps certain Jacobi Poincaré series to half-integral weight Poincaré series and we construct a subspace of Jacobi cusp forms on which the map $\mathcal{Z}_M$ is injective. By using the above results we relate the Fourier coefficients of certain Jacobi cusp forms to central values of the twisted modular L-functions and we improve certain non-vanishing results for Jacobi Poincaré series established by S. Das.

1. Introduction

In [14], Skoruppa and Zagier constructed certain lifting maps between the space of Jacobi cusp forms of integer index and a subspace of cusp forms of integral weight.

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In [7], Gross, Kohnen and Zagier constructed kernel functions for these liftings and by using these correspondences, they obtained deep formulas relating the height pairings of Heegner points to the Fourier coefficients of Jacobi cusp forms. Further they relate the latter object to the central critical values of twisted L-functions of Hecke eigen cusp forms. In [6], Eichler and Zagier considered a map between Jacobi forms of integer index and half-integral weight modular forms which gives a perfect isomorphism in the index 1 case. This isomorphism is an important step to complete the Saito-Kurokawa correspondence. In [9], the authors studied Eichler–Zagier map $\mathcal{Z}_m$ on the space of Jacobi cusp forms of higher level and integer index m and constructed a subspace of the latter, called the newform space, isomorphic to certain subspace of half-integral weight cusp forms under the map $\mathcal{Z}_m$. Further they discuss interesting consequences of the developed theory. A natural question is to what extent this theory can be generalized for matrix index Jacobi cusp forms.

As a first step in this direction, Bringmann [3] constructed lifting maps between the space of Jacobi cusp forms of matrix index and the space of cusp forms of integral weight in the case the order of the matrix is congruent to 1 mod 8, by following the methods used in [7]. In this paper we define an Eichler–Zagier map $\mathcal{Z}_M$ between the space of Jacobi cusp forms of matrix index M and the space of half-integral weight cusp forms and discuss mapping properties of $\mathcal{Z}_M$. In the case the order of M is congruent to 1 mod 8 we show that $\mathcal{Z}_M$ maps the Jacobi Poincaré series $P_{D,r}, D \equiv \square \pmod{2 \det 2M}, \gcd(D, \det 2M) = 1$, to a certain half-integral weight Poincaré series in the plus space. Further we construct a subspace of Jacobi cusp forms on which the map $\mathcal{Z}_M$ is injective and we call it the newform space. By using the above results we relate the Fourier coefficients of the newforms to central values of the twisted modular L-functions and also we produce certain non-vanishing results for Jacobi Poincaré series one of which improves the non-vanishing condition established in [5].

This article is organized as follows. In the next section, we recall the action of Hecke operators on the space of Jacobi cusp forms of matrix index, the Fourier expansion of Jacobi Poincaré series. We also recall the Fourier expansion of half-integral weight Poincaré series. In §3, we define an Eichler–Zagier map and show that it maps Jacobi cusp forms of matrix index to half-integral weight cusp forms. Throughout §4 and §5, we assume that the order of the matrix index is congruent to 1 mod 8. In §4, we show that a certain Jacobi Poincaré series is mapped to half-integral weight Poincaré series under the map defined in §3. Further we construct a subspace of Jacobi cusp forms, call the newform space, on which the Eichler–Zagier map is injective. Using these results we relate the Fourier coefficients of certain Jacobi cusp forms to the central values of twisted modular L-functions. In §5, we provide certain non-vanishing conditions for Jacobi Poincaré series as a consequence of the mapping properties discussed in §4.

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