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Spectral continuity for aperiodic quantum systems I. General theory ☆

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ABSTRACT

How does the spectrum of a Schrödinger operator vary if the corresponding geometry and dynamics change? Is it possible to define approximations of the spectrum of such operators by defining approximations of the underlying structures? In this work a positive answer is provided using the rather general setting of groupoid C^*-algebras. A characterization of the convergence of the spectra by the convergence of the underlying structures is proved. In order to do so, the concept of continuous field of groupoids is slightly extended by adding continuous fields of cocycles. With this at hand, magnetic Schrödinger operators on dynamical systems or Delone systems fall into this unified setting. Various approximations used in computational physics, like the periodic or the finite cluster approximations, are expressed through the tautological groupoid, which provides a universal model for fields of groupoids. The use of the Hausdorff topology turns out

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to be fundamental in understanding why and how these approximations work.

Contents

1. Introduction and main results

The present paper delivers a characterization of the convergence of the spectrum of Schrödinger-type operators via the convergence of the underlying structures equipped with a suitable topology. This is the first paper of a series of articles aiming at providing a method to compute the spectrum of a self-adjoint operator on a Hilbert space. Such