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# Nonlocal problems with critical Hardy nonlinearity <sup>☆</sup>

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## ABSTRACT

By means of variational methods we establish existence and multiplicity of solutions for a class of nonlinear nonlocal problems involving the fractional  $p$ -Laplacian and a combined Sobolev and Hardy nonlinearity at subcritical and critical growth.

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## 1. Introduction

Let  $\Omega \subset \mathbb{R}^N$  be a smooth open bounded set containing 0. In this work we study the existence and multiplicity of solutions to the following nonlocal problem driven by the fractional  $p$ -Laplacian operator

$$\begin{cases} (-\Delta_p)^s u = \lambda |u|^{r-2} u + \mu \frac{|u|^{q-2} u}{|x|^\alpha} & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \quad (1.1)$$

being  $0 < s < 1$ ,  $p > 1$ ,  $\lambda, \mu > 0$ ,  $0 \leq \alpha \leq ps < N$ ,  $p \leq r \leq p^*$ ,  $p \leq q \leq p_\alpha^*$ , where

$$p^* := \frac{Np}{N - ps}, \quad p_\alpha^* := \frac{(N - \alpha)p}{N - ps}, \quad (1.2)$$

are the fractional critical *Sobolev* and *Hardy-Sobolev* exponents respectively. The fractional  $(p, s)$ -Laplacian operator  $(-\Delta_p)^s$  is the differential of the convex functional

$$u \mapsto \frac{1}{p} [u]_{s,p}^p := \frac{1}{p} \int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^p}{|x - y|^{N+ps}} dx dy$$

defined on the Banach space (with respect to the norm  $[u]_{s,p}$  defined above)

$$W_0^{s,p}(\Omega) := \{u \in L_{\text{loc}}^1(\mathbb{R}^N) : u \equiv 0 \text{ in } \mathbb{R}^N \setminus \Omega \text{ and } [u]_{s,p} < +\infty\}.$$

This definition is consistent, up to a normalization constant, with the linear fractional Laplacian  $(-\Delta)^s$  for the case  $p = 2$ . The exponents in (1.2) arise from the general fractional Hardy-Sobolev inequality

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