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# Nonlocal problems with critical Hardy nonlinearity ☆

Wenjing Chen [a], Sunra Mosconi [b], Marco Squassina [c,*]

[a] *School of Mathematics and Statistics, Southwest University Chongqing 400715, People's Republic of China*
[b] *Dipartimento di Matematica e Informatica, Università degli Studi di Catania, Viale A. Doria 6, 95125 Catania, Italy*
[c] *Dipartimento di Matematica e Fisica, Università Cattolica del Sacro Cuore, Via dei Musei 41, 25121 Brescia, Italy*

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By means of variational methods we establish existence and multiplicity of solutions for a class of nonlinear nonlocal problems involving the fractional $p$-Laplacian and a combined Sobolev and Hardy nonlinearity at subcritical and critical growth.



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* Corresponding author.
*E-mail addresses:* wjchen@swu.edu.cn (W. Chen), mosconi@dmi.unict.it (S. Mosconi), marco.squassina@unicatt.it (M. Squassina).

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**Contents**



## 1. Introduction

Let $\Omega \subset \mathbb{R}^N$ be a smooth open bounded set containing 0. In this work we study the existence and multiplicity of solutions to the following nonlocal problem driven by the fractional $p$-Laplacian operator

$$\begin{cases} (-\Delta_p)^s u = \lambda |u|^{r-2}u + \mu \frac{|u|^{q-2}u}{|x|^\alpha} & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{1.1}$$

being $0 < s < 1$, $p > 1$, $\lambda, \mu > 0$, $0 \le \alpha \le ps < N$, $p \le r \le p^*$, $p \le q \le p^*_\alpha$, where

$$p^* := \frac{Np}{N - ps}, \qquad p^*_\alpha := \frac{(N-\alpha)p}{N - ps}, \tag{1.2}$$

are the fractional critical *Sobolev* and *Hardy–Sobolev* exponents respectively. The fractional $(p, s)$-Laplacian operator $(-\Delta_p)^s$ is the differential of the convex functional

$$u \mapsto \frac{1}{p}[u]^p_{s,p} := \frac{1}{p} \int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^p}{|x - y|^{N+ps}}\, dx\, dy$$

defined on the Banach space (with respect to the norm $[u]_{s,p}$ defined above)

$$W^{s,p}_0(\Omega) := \left\{ u \in L^1_{\text{loc}}(\mathbb{R}^N) : u \equiv 0 \text{ in } \mathbb{R}^N \setminus \Omega \text{ and } [u]_{s,p} < +\infty \right\}.$$

This definition is consistent, up to a normalization constant, with the linear fractional Laplacian $(-\Delta)^s$ for the case $p = 2$. The exponents in (1.2) arise from the general fractional Hardy–Sobolev inequality

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