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BOHR'S PHENOMENON FOR FUNCTIONS ON THE BOOLEAN CUBE

ANDREAS DEFANT, MIECZYSLAW MASTYŁO, AND ANTONIO PÉREZ

ABSTRACT. We study the asymptotic decay of the Fourier spectrum of real functions $f: \{-1, 1\}^N \rightarrow \mathbb{R}$ in the spirit of Bohr's phenomenon from complex analysis. Every such function admits a canonical representation through its Fourier-Walsh expansion $f(x) = \sum_{S \subset \{1, \dots, N\}} \widehat{f}(S) x^S$, where $x^S = \prod_{k \in S} x_k$. Given a class \mathcal{F} of functions on the Boolean cube $\{-1, 1\}^N$, the Boolean radius of \mathcal{F} is defined to be the largest $\rho \geq 0$ such that $\sum_S |\widehat{f}(S)| \rho^{|S|} \leq \|f\|_\infty$ for every $f \in \mathcal{F}$. We give the precise asymptotic behavior of the Boolean radius of several natural subclasses of functions on finite Boolean cubes, as e.g. the class of all real functions on $\{-1, 1\}^N$, the subclass made of all homogeneous functions or certain threshold functions. Compared with the classical complex situation subtle differences as well as striking parallels occur.

1. INTRODUCTION

Recent years show a remarkable increase of articles on Bohr's phenomenon in complex analysis. The primal Bohr's famous power series theorem from [5] states that $r = 1/3$ is the largest $r > 0$ satisfying that for every complex polynomial $\sum_k c_k z^k$

$$\sup_{z \in r\mathbb{D}} \sum_k |c_k z^k| \leq \sup_{z \in \mathbb{D}} \left| \sum_k c_k z^k \right|$$

Extensions of Bohr's inequality have been later studied for other classes of functions and domains, see e.g. the recent survey [1]. Dineen and Timoney [12] considered this issue in the multidimensional setting, and Boas and Khavinson [4] later introduced for $N \in \mathbb{N}$ the N -th Bohr radius K_N as the largest $r > 0$ such that for every complex polynomial $\sum_{\alpha \in \mathbb{N}_0^N} c_\alpha z^\alpha$ in N variables

$$(1) \quad \sup_{z \in r\mathbb{D}^N} \sum_{\alpha \in \mathbb{N}_0^N} |c_\alpha z^\alpha| \leq \sup_{z \in \mathbb{D}^N} \left| \sum_{\alpha \in \mathbb{N}_0^N} c_\alpha z^\alpha \right|$$

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