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Andreas Defant, Mieczysław Mastyło, Antonio Pérez


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# BOHR'S PHENOMENON FOR FUNCTIONS ON THE BOOLEAN CUBE

ANDREAS DEFANT, MIECZYSŁAW MASTYŁO, AND ANTONIO PÉREZ

ABSTRACT. We study the asymptotic decay of the Fourier spectrum of real functions $f\colon \{-1,1\}^N \longrightarrow \mathbb{R}$ in the spirit of Bohr's phenomenon from complex analysis. Every such function admits a canonical representation through its Fourier-Walsh expansion $f(x) = \sum_{S\subset\{1,\dots,N\}} \widehat{f}(S)x^S$, where $x^S = \prod_{k\in S} x_k$. Given a class $\mathcal{F}$ of functions on the Boolean cube $\{-1,1\}^N$, the Boolean radius of $\mathcal{F}$ is defined to be the largest $\rho \geq 0$ such that $\sum_S |\widehat{f}(S)|\rho^{|S|} \leq \|f\|_\infty$ for every $f \in \mathcal{F}$. We give the precise asymptotic behavior of the Boolean radius of several natural subclasses of functions on finite Boolean cubes, as e.g. the class of all real functions on $\{-1,1\}^N$, the subclass made of all homogeneous functions or certain threshold functions. Compared with the classical complex situation subtle differences as well as striking parallels occur.

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## 1. Introduction

Recent years show a remarkable increase of articles on Bohr's phenomenon in complex analysis. The primal Bohr's famous power series theorem from [5] states that $r = 1/3$ is the largest $r > 0$ satisfying that for every complex polynomial $\sum_k c_k z^k$

$$\sup_{z\in r\mathbb{D}} \sum_k |c_k z^k| \leq \sup_{z\in\mathbb{D}} \left|\sum_k c_k z^k\right|$$

Extensions of Bohr's inequality have been later studied for other classes of functions and domains, see e.g. the recent survey [1]. Dineen and Timoney [12] considered this issue in the multidimensional setting, and Boas and Khavinson [4] later introduced for $N \in \mathbb{N}$ the $N$-th Bohr radius $K_N$ as the largest $r > 0$ such that for every complex polynomial $\sum_{\alpha\in\mathbb{N}_0^N} c_\alpha z^\alpha$ in $N$ variables

$$\sup_{z\in r\mathbb{D}^N} \sum_{\alpha\in\mathbb{N}_0^N} |c_\alpha z^\alpha| \leq \sup_{z\in\mathbb{D}^N} \left|\sum_{\alpha\in\mathbb{N}_0^N} c_\alpha z^\alpha\right| \tag{1}$$

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