Accepted Manuscript

Factorization of the identity through operators with large diagonal

Niels Jakob Laustsen, Richard Lechner, Paul F.X. Müller

PII: S0022-1236(18)30069-7
DOI: https://doi.org/10.1016/j.jfa.2018.02.010
Reference: YJFAN 7965

To appear in: *Journal of Functional Analysis*

Received date: 30 November 2017
Accepted date: 19 February 2018

Please cite this article in press as: N.J. Laustsen et al., Factorization of the identity through operators with large diagonal, *J. Funct. Anal.* (2018), https://doi.org/10.1016/j.jfa.2018.02.010

FACTORIZATION OF THE IDENTITY THROUGH OPERATORS WITH LARGE DIAGONAL

NIELS JAKOB LAUSTSEN, RICHARD LECHNER, AND PAUL F.X. MÜLLER

Abstract. Given a Banach space X with an unconditional basis, we consider the following question: does the identity operator on X factor through every operator on X with large diagonal relative to the unconditional basis? We show that on Gowers' unconditional Banach space, there exists an operator for which the answer to the question is negative. By contrast, for any operator on the mixed-norm Hardy spaces $H^p(H^q)$, where $1 \le p, q < \infty$, with the bi-parameter Haar system, this problem always has a positive solution. The spaces $L^p, 1 < p < \infty$, were treated first by Andrew [*Studia Math.* 1979].

1. Introduction

Let X be a Banach space. A *basis* for X will always mean a Schauder basis. We denote by I_X the identity operator on X, and write $\langle \cdot, \cdot \rangle$ for the bilinear duality pairing between X and its dual space X^*. By an *operator* on X, we understand a bounded and linear mapping from X into itself.

Suppose that X has a normalized basis $(b_n)_{n \in \mathbb{N}}$, and let $b_n^* \in X^*$ be the n^{th} coordinate functional. For an operator T on X, we say that:

▷ *T has large diagonal* if $\inf_{n \in \mathbb{N}} |\langle T b_n, b_n^* \rangle| > 0$;
▷ T is *diagonal* if $\langle T b_m, b_n^* \rangle = 0$ whenever $m, n \in \mathbb{N}$ are distinct;
▷ *the identity operator on X factors through T* if there are operators R and S on X such that the diagram

$$\begin{array}{ccc} X & \xrightarrow{\;I_X\;} & X \\ {\scriptstyle R}\big\downarrow & & \big\uparrow{\scriptstyle S} \\ X & \xrightarrow[\;T\;]{} & X \end{array}$$

is commutative.

Suppose that the basis $(b_n)_{n \in \mathbb{N}}$ for X is unconditional. Then the diagonal operators on X correspond precisely to the elements of $\ell_\infty(\mathbb{N})$, and so for each operator T on X with large diagonal, there is a diagonal operator S on X such that $\langle ST b_n, b_n^* \rangle = 1$ for each $n \in \mathbb{N}$. This observation naturally leads to the following question.

Question 1.1. *Can the identity operator on X be factored through each operator on X with large diagonal?*

In classical Banach spaces such as ℓ^p with the unit vector basis and L^p with the Haar basis, the answer to this question is known to be positive. These are the

Date: March 1, 2018.
2010 *Mathematics Subject Classification.* Primary: 30H10, 47A68, 47B37. Secondary: 46B15, 46B25, 47A53 .
Key words and phrases. Factorization of operators, mixed-norm Hardy spaces, Fredholm theory, Gowers-Maurey spaces.
The research of Lechner and Müller is supported by the Austrian Science Foundation (FWF) Pr.Nr. P23987 P22549.