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Linear hydrodynamic modelling of arrays of submerged oscillating cylinders

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ABSTRACT

The radiation and diffraction problems for an array of submerged circular cylinders with vertical axes of revolution are formulated exactly to first order in the frequency domain. Matched eigenfunction expansions and a transform matrix method are used to solve the scattering problem for an arbitrary array using a truncated system of matrix equations. For a single shallowly submerged cylinder, the model is used to characterise the occurrence of resonances in the region of fluid above the cylinder moving in heave and surge motion. The method is then applied to a square array of four cylinders, and the effect of array interactions demonstrated. It is found that fluid resonances above the cylinder are still important, but are modified by multiple scattering. Finally, the mean vertical drift force is calculated from the first order solution by direct pressure integration over the body surface.

1. Introduction

Some wave energy devices and offshore structures can be modelled as submerged oscillating circular cylinders with vertical axes of revolution. This paper presents a semi-analytical method to model an array of such structures using linear theory, in which the multiple body scattering problem is solved by the use of a transform matrix and a system of matrix equations. The use of such a transform matrix method requires only that the diffraction and radiation problem be formulated exactly for a single cylinder; the complete array solution can then be solved simultaneously and exactly. This type of model can be used to analyse the hydrodynamics of large arrays and offers some advantages over traditional boundary element methods when changing the cylinder or array geometry as re-meshing is not required.

This work is partly motivated by the wave energy device under development by Carnegie Clean Energy, the CETO wave energy converter. This device consists of a shallowly submerged cylinder tethered to the sea floor, see Fig. 1. The CETO device may be installed in arrays with relatively small spacing; we must therefore consider the effects of device interactions when modelling the hydrodynamics of such an array as this may be important for design of the system for optimum power production (e.g. Borgarino et al. [1]).

A submerged oscillating cylinder has significantly altered hydrodynamic properties compared to the same structure floating at the surface due to the layer of fluid above the cylinder. The phenomenon of negative added mass can be observed for very shallow submergences and resonances in the region of fluid above the cylinder can produce large peaks in the damping, as observed by Newman et al. [3] and McIver and Evans [4] for the 2-dimensional and 3-dimensional radiation problems respectively and Martin and Farina [5] for an oscillating submerged plate.

Linear modelling of diffraction around cylinders under the influence of waves has been the focus of much previous work as many marine structures are (at least approximately) cylindrical, for example, the legs of a tension-leg or semi-submersible oil and gas platform. The linear diffraction problem for a single surface piercing truncated cylinder was solved by Garrett [6] using an eigenfunction expansion method and Yeung [7] solved the complementary radiation problems in surge, heave and pitch. The radiation and diffraction problems for a single submerged cylinder were solved by Jiang et al. [8,9].

The array diffraction problem has also been extensively studied. Kagemoto and Yue [10] combined features of the matrix method of Spring and Monkmeyer [11] and Simon [12], and the multiple scattering technique of Twersky [13] and Ohkusu [14] to solve the complete scattering problem for an array of arbitrary bodies. They represented the scattered wavefield around each body as a summation of cylindrical waves with undetermined amplitudes. A set of linear equations were derived to satisfy the diffraction characteristics of all the bodies. This system was then solved simultaneously for all of the unknown amplitude coefficients. The solution is in principle exact (within the context of linearised theory).

Linton and Evans [15] solved the diffraction problem for an array of N bottom mounted cylinders using the method of Spring and Monkmeyer [11], and Kim [16] extended this to the complementary radiation problem. Yilmaz and Incecik [17] applied the interaction theory of Kagemoto and Yue [10] to the diffraction problem for truncated

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Fig. 1. Carnegie Clean Energy CETO wave energy devices. From [2].

cylinders and Yilmaz [18] calculated the added mass and damping of an array of bodies oscillating as one. The present work is based on the methods outlined by Siddorn and Eatock Taylor [19], who investigated the wave excitation and response of surface-piercing truncated cylinders which are free to oscillate independently. They used the theory of Kagemoto and Yue [10], and developed an extension of this for the full radiation problem. In this work we adapt the method of Siddorn and Eatock Taylor [19] to consider submerged cylinders and produce results for a simple array. Array problems for different structures continue to be of interest – consider, for example, Chatjigeorgiou and Katsardi [20].

It is well known that linear solutions provide the basis for calculating mean drift loads in regular waves, which represent a simple, but sometimes important, non-linear effect. The second order mean drift forces on a two dimensional submerged horizontal cylinder were obtained by Ogilvie [21] using the first order solution and integrating the pressure over the surface. Lee and Newman [22] derived expressions for the vertical drift force on a submerged body assuming it is geometrically slender with respect to the body length, wavelength and submergence and using Kochin functions which avoid the need to determine the pressure distribution on the body surface. Mavrakos [23] solved for the mean vertical drift forces on axisymmetric bodies using conservation of momentum within a control volume. Here we solve for the second order mean vertical drift force by numerical pressure integration over the body surface as an example of further use of the model outputs in calculating hydrodynamic forces.

In Section 2 we present the diffraction and radiation solutions for a single submerged circular cylinder, followed by the transform matrix method to solve the system for an arbitrary array of cylinders of varying submergence, radius and thickness. In Section 3 the model is validated through comparison to the Hydrostar boundary element (BEM) software. Some limitations of the model and resonant behaviours above the cylinder are discussed, followed by array damping and exciting force results for a simple four cylinder array.

2. Potential flow solutions for a submerged cylinder

2.1. Preliminaries

We use linear potential flow theory; the wave amplitudes and body motions are small compared to the wavelength, device size and submergence. We use a velocity potential, denoted Φ, throughout the fluid that varies sinusoidally with frequency ω. The potential can be written in cylindrical coordinates as a complex Fourier series:

$$\Phi = \Phi(r, \theta, z, t) = \mathrm{Re}\{\phi(r, \theta, z)\mathrm{e}^{\mathrm{i}\omega t}\} = \mathrm{Re}\left\{\mathrm{e}^{\mathrm{i}\omega t}\sum_{f=-\infty}^{\infty}\mathrm{e}^{\mathrm{i}f\theta}\chi_f(r, z)\right\}. \tag{1}$$

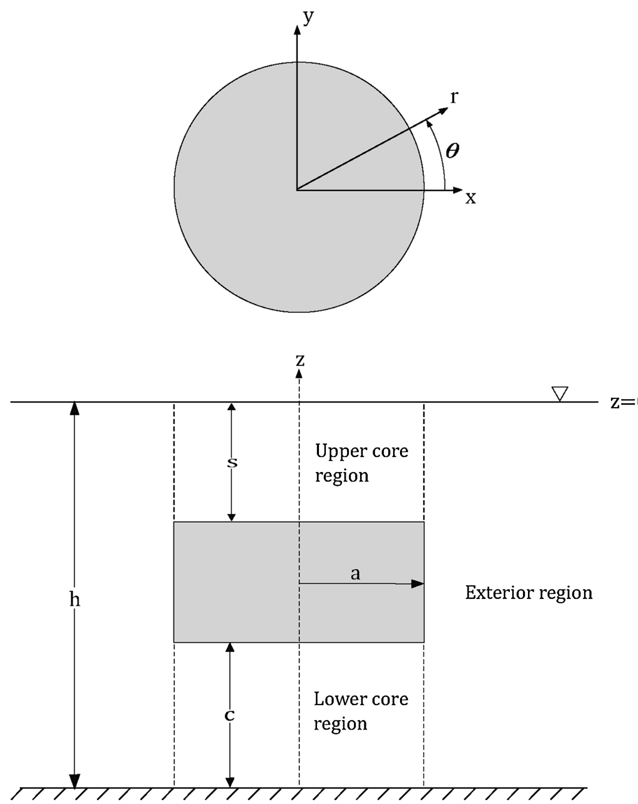

Fig. 2. Definition sketch.

Fig. 2 defines the coordinate system, t is time and ϕ is the complex time-invariant velocity potential. The angle invariant potential for each Fourier mode f is denoted χ_f. The fluid is divided into three regions, the core regions above and below the cylinder and an exterior region. The cylinder radius is a, the water depth h, the distance from the free surface to the top of the cylinder is submergence s and the clearance between the sea floor and the bottom of the cylinder is c.

The velocity potential must satisfy the Laplace equation in the fluid:

$$\nabla^2\phi = 0, \tag{2}$$

and the linearised boundary conditions on the seabed, free surface and cylinder surface (S) respectively:

$$\left.\frac{\partial\phi}{\partial z}\right|_{z=-h} = 0, \tag{3}$$

$$\left[-\omega^2\phi + g\frac{\partial\phi}{\partial z}\right]_{z=0} = 0, \tag{4}$$

$$\left.\frac{\partial\phi}{\partial n}\right|_{S} = u_n, \tag{5}$$

where u_n is the normal component of the complex amplitude of the cylinder velocity at any point on its surface.

The dispersion relation arising from the free surface boundary condition (4) in finite depth is given by:

$$\omega^2 = \mathrm{g}k_m\tanh(k_m h). \tag{6}$$

The positive real solution, k_0, is the wavenumber of the propagating mode. The negative imaginary solutions, k_m for $m = 1, 2, \ldots$ are the wavenumbers of the evanescent modes. The evanescent waves are non-propagating modes, which decay exponentially with distance from the cylinder and as such cannot transfer energy into the far field.