Applied Ocean Research 81 (2018) 47–71

Contents lists available at ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

# High performance analysis of lateral sloshing response in vertical cylinders with dual circular or arc-shaped porous structures

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ARTICLE INFO

*Keywords:*
Liquid sloshing
Scaled boundary finite element method
Linear potential theory
Cylindrical tank
Porous structures

ABSTRACT

This study presents a semi-analytical solution with the scaled boundary finite element method (SBFEM) for the sloshing of liquid in vertical cylinders with the coaxial dual circular or arc-shaped porous structures in the context of the linear potential theory. Firstly, the whole flow domain can be divided into three sub-domains with the porous structures and the wall of tank, in which the arc-shaped porous structure should be extended to the whole circle and the perforated coefficient of the extension section is set to infinite. Then the velocity potential of flow in each sub-domain is determined by utilizing the SBFEM for solving the Helmholtz equation or the modified Helmholtz equations, and the fundamental equations and the solution process of the SBFEM for this sloshing problem have been derived in detail. In the SBFEM, one of the superior features is that only the circumferential boundary of the computational domain needs to be discretized with the same form of the finite element. As a result, the spatial dimension of the problem can be reduced by one. As a key point, although there are the dual circular or arc-shaped porous structures, only the wall of tank should be discretized. Meanwhile, the governing equations can be solved analytically in the radial direction by using the Bessel functions or the modified Bessel functions. Numerical examples show that only a small quantity of the discretized elements over the cylindrical boundary can achieve an excellent agreement between the numerical results obtained by the proposed method and the analytical solutions, which demonstrate the accuracy and efficiency of this formulation. Finally, several numerical examples are further studied to investigate the effects of the radius of the porous structures, the porous-effect parameter, the nondimensional wave number, and the opening degree together with the distribution of the centre of the arc-shaped porous structures on the sloshing characteristics of liquid in the cylindrical container with the porous structures.

## 1. Introduction

Recently, there is an increasing interest in the sloshing problem of liquid in a container because of its wide use in many fields, such as aerospace, hydraulic, waterway and ocean, civil and transport engineering. In the tanks with liquid sloshing, severe hydrodynamic loads may generate under the external excitation, which may bring out threat to the structural integrity and stability of the systems [1]. One of the effective ways to control the sloshing characteristics or to reduce the sloshing force acting on the liquid tank is installing the solid or porous baffle inside the container.

In recent decades, a large number of works which refer to the liquid sloshing in tanks with solid or porosity baffles have been investigated theoretically, experimentally and numerically by many researchers. For example, Maleki and Ziyaeifar [2] investigated the impact of the horizontal and vertical baffles on the damping ratio of the liquid sloshing in the circular-cylindrical tanks by solving the Laplace's differential equation analytically, and further compared the solutions with a series of experiments. Faltinsen et al. [3,4] developed an analytical model to describe the two-dimensional sloshing characteristics in a rectangular baffled tank by applying the linear potential-flow theory, and the similar works have also been studied by Goudarzi et al. [5]. By the means of separation of variables and the superposition principle, Wang et al. [6] obtained the sloshing frequencies and the corresponding modes of liquid in the rigid cylindrical baffled tanks, and on the basis of this model, they further employed this method to study the sloshing behaviours of flow in the rigid cylindrical multiple baffled tanks under the lateral excitation [7]. Choudhary and Bora [1] carried out an analytical solution for the sloshing frequencies and modes in a cylindrical storage container whose free liquid surface was partly covered with an

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https://doi.org/10.1016/j.apor.2018.09.017
Received 4 July 2018; Received in revised form 31 August 2018; Accepted 25 September 2018

annular baffle, and it was found that the natural frequencies of system increases significantly as the width of the baffle increases.

For the aspects of the experimental investigation, some significant work has also been reported in the literature. Panigrahy et al. [8] conducted a series of experimental studies on sloshing behaviours of liquid in the baffled square tanks under the horizontal movement. In this study, the pressure and the free surface elevation of water were measured, and the results showed that the sloshing effect decreased significantly by introducing the baffles in the tank. Akyildiz et al. [9] studied experimentally the sloshing responses of liquid in a cylindrical storage container with the ring baffles subjected to the excitation of roll motion. They further investigated the non-linear behaviours of liquid and the corresponding damping characteristics of the ring baffles by measuring the wave elevation and the pressure values on the tank wall, and it can be concluded that the sloshing loads were reduced effectively by arranging the ring baffle. The sloshing characteristics of the ideal liquid in the rectangular containers with a perforated baffle have been presented both experimentally and numerically by Molin and Remy [10]. Under the horizontal and rolling motions, they estimated the hydrodynamic coefficients and the free surface elevation at the wall, and further showed a comparison between the experimental and numerical results. Jin et al. [11] designed a series of experimental tests to study the characteristics of a horizontal perforated plate in a rectangular liquid tank subjected to the horizontal excitation. The results showed that the perforated plates played an important role in restraining violent sloshing effects and the plates were minimally influenced by both the first-order and third-order models. Xue et al. [12] developed an experimental investigation of liquid sloshing in the rectangular containers with several types of the vertical baffles under the horizontal excitation. In this study, the hydrodynamic pressures acting on the baffle and tank wall were carried out, and the results presented that the sloshing pressure in the baffles not only depended on the relations between excitation frequency and natural frequency of the system but also concerned with the configuration and the location of baffles.

At the same time, many numerical methods are also widely applied to predict the sloshing phenomena of liquid in the baffled tanks. Among them, the finite element method (FEM) and the boundary element method (BEM) are the most popular method for the investigation of the sloshing problems. For example, based on the coupled structural-acoustic FEM, Cho et al. [13] investigated the free-vibration behaviours of the liquid sloshing in the cylindrical liquid-storage tanks with the annular baffles. In addition, Cho et al. [14] extended their work to study the sloshing dynamic response of liquid in the moving cylindrical fuel-storage tanks with the disc-type elastic baffles by the means of the ALE finite element method. Furthermore, as the further extension of the previous works, they investigated the resonance sloshing characteristics of liquid in a 2D baffled rectangular tank by utilizing the FEM [15]. Alfredo et al. [16] provided a FEM model for the sloshing modes in a rectangular liquid tank with the elastic plates, and introduced the Reissner–Mindlin equations to model the deformation of the elastic plates. Biswal et al. [17,18] developed a finite element method to analyze the dynamic response characteristics of liquid sloshing in cylindrical containers with annular baffles. By using the finite difference method, they further studied the sloshing problem by employing both the Galerkin method and the fourth-order Runge–Kutta method. They also investigated the non-linear characteristics of the liquid sloshing in a rigid rectangular container with the rigid baffles under the translational base excitation [19]. Zheng et al. [20] developed a FLUENT model to study the sloshing of liquid in the longitudinal tanks with different baffles including the one with small holes. By the means of the Galerkin finite element method, Kumar et al. [21] studied the time domain dynamic sloshing response behaviours of liquid in a two-dimensional rectangular container with the perforated baffles. By comparing with the solid baffles, they found that the perforated baffles were more effective to improve hydrodynamic characteristics of system. In addition, the FEM has also been proposed to investigate the sloshing effect of the perforated baffle in the 3-D trapezoidal fuel tank by Mohan [22]. For the aspects of the BEM, Gedikli et al. [23] studied the seismic response of the liquid sloshing in the cylindrical tanks with a rigid annular baffle by using the BEM. Furthermore, by utilizing the variational boundary element method, Gedikli et al. [24] further extended their work to evaluate the liquid sloshing phenomenon in the cylindrical baffled containers. On the basis of the potential flow theory, Firouz-Abadi et al. [25,26] proposed a 3-D BEM model to simulate the sloshing of liquid in the baffled containers with arbitrary geometries. By employing the multi-domain BEM, He et al. [27] analyzed the nonlinear sloshing phenomena in both two-dimensional and three-dimensional liquid containers with the porosity girder in time domain. Under the assumptions of the inviscid and incompressible fluid, Sygulski [28] obtained the natural sloshing frequencies and the corresponding modes of liquid in the three-dimensional baffled containers by the means of the BEM. Based on the BEM formulation, Ebrahimian et al. [29] developed the 2-D and axisymmetric model of a container with multiple baffles to obtain equivalent mechanical model (EMM) of the liquid sloshing. Kolaei et al. [30] used the BEM for the investigation of the liquid sloshing in the partly-filled cylindrical tanks with different baffles under the laterally excitation. Except for the FEM and the BEM, many others valuable numerical methods have also been widely employed to investigate the sloshing of liquid in the baffled tanks, such as the finite volume method [31,32], the two-step projection method [33], the smoothed particle hydrodynamics (SPH) method [34], the multi-modal method [35], the virtual boundary force (VBF) method [36], and so on.

In recent years, the liquid sloshing problems have been resoundingly investigated by a novel numerical technique named the scaled boundary finite element method (SBFEM). The SBFEM was developed by Wolf and Song [37–40] in the 1990s and it has many superior properties unlike the FEM and the BEM, such as, the solutions can be obtained analytically in the radial direction of the computational domain, no fundamental solutions are required compared with the BEM, and it is effective for the analysis of the problems in the unbounded domains and relating to stress singularities, and so on. Owing to these reasons, the SBFEM is widely used not only in the analysis of the soil-structure interaction [41,42], geotechnical structures [43–47], dam-water-foundation interaction problems [48,49], elastic waveguide [50], heat conduction [51,52], frictional contact problems [53], fracture mechanics [54–58], seepage problems [59–61], electromagnetic [62,63], fluid mechanics [64–68], etc., but also in the liquid sloshing problem. For example, Teng et al. [69] developed the SBFEM model to simulate the water sloshing in a rectangular tank. Wang et al. [70–73] firstly extended the SBFEM with very few elements to study some interesting and complex sloshing problems with various baffles including the liquid sloshing in the elliptical T-shaped baffled containers in frequency and time domain (including the transverse and longitudinal excitation), and the sloshing of liquid in cylindrical tanks with multi baffles under the lateral excitation. Meanwhile, the liquid sloshing in the three dimensional toroidal tank having the complex geometry with various baffles under the lateral excitation has also been studied by Wang et al. [74]. From the references about the sloshing of liquid, it can be illustrated that the perforated baffle may be much more powerful to suppress the sloshing force or maintain the stability of the system in some cases. However, there are relatively few studies about the liquid sloshing with the porous baffle, and in the current research work of which, most of them are limited to the rectangle model, the liquid sloshing problem in the cylindrical tank with the porous structures is rarely involved.

For the investigation of the cylindrical porous structure, a similar problem of short crested wave with the uniform or nonuniform porous cylinder was considered by Tao et al. [66,67], in which the SBFEM was also introduced. In this paper, on the basis of the previous works which were presented in the papers [66,67], the SBFEM is extended to study the liquid sloshing in a cylindrical tank with the coaxial dual circular or