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Nonparametric inference of gradual changes in the jump behaviour of time-continuous processes

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Abstract

In applications the properties of a stochastic feature often change gradually rather than abruptly, that is: after a constant phase for some time they slowly start to vary. In this paper we discuss statistical inference for the detection and the localization of gradual changes in the jump characteristic of a discretely observed Ito semimartingale. We propose a new measure of time variation for the jump behaviour of the process. The statistical uncertainty of a corresponding estimate is analysed by deriving new results on the weak convergence of a sequential empirical tail integral process and a corresponding multiplier bootstrap procedure.

MSC: 60F17; 60G51; 62G10

Keywords: Lévy measure; Jump compensator; Transition kernel; Empirical processes; Weak convergence; Gradual changes

1. Introduction

Stochastic processes in continuous time are widely used in the applied sciences nowadays, as they allow for a flexible modelling of the evolution of various real-life phenomena over time. Speaking of mathematical finance, of particular interest is the family of semimartingales,

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which is theoretically appealing as it satisfies a certain condition on the absence of arbitrage in financial markets and yet is rich enough to reproduce stylized facts from empirical finance such as volatility clustering, leverage effects or jumps. For this reason, the development of statistical tools modelled by discretely observed Itō semimartingales has been a major topic over the last years, both regarding the estimation of crucial quantities used for model calibration purposes and with a view on tests to check whether a certain model fits the data well. For a detailed overview of the state of the art we refer to the recent monographs by Jacod and Protter [18] and Aït-Sahalia and Jacod [1].

These statistical tools typically differ highly, depending on the quantities of interest. When the focus is on the volatility, most concepts are essentially concerned with discrete observations of the continuous martingale part. In this case one is naturally close to the Gaussian framework, and so a lot of classical concepts from standard parametric statistics turn out to be powerful methods. The situation is different with a view on the jump behaviour of the process, mainly for two reasons: On one hand there is much more flexibility in the choice of the jump measure than there is regarding the diffusive part. On the other hand even if one restricts the model to certain parametric families the standard situation is the one of β-stable processes, $0 < \beta < 2$, for which the mathematical analysis is quite difficult, at least in comparison to Brownian motion. To mention recent works besides the afore-mentioned monographs, see for example [24] and [15] on the estimation of the jump distribution function of a Lévy process or Todorov [28] on the estimation of the jump activity index from high-frequency observations.

In the following, we are interested in the evolution of the jump behaviour over time in a completely non-parametric setting where we assume only structural conditions on the characteristic triplet of the underlying Itō semimartingale. To be precise, let $X = (X_t)_{t\geq 0}$ be an Itō semimartingale with a decomposition

$$X_t = X_0 + \int_0^t b_s\,ds + \int_0^t \sigma_s\,dW_s + \int_0^t \int_{\mathbb{R}} u 1_{\{|u|\leq 1\}}(\mu - \bar{\mu})(ds, du) + \int_0^t \int_{\mathbb{R}} u 1_{\{|u|>1\}}\mu(du, dz), \tag{1.1}$$

where W is a standard Brownian motion, μ is a Poisson random measure on $\mathbb{R}^+ \times \mathbb{R}$, and the predictable compensator $\bar{\mu}$ satisfies $\bar{\mu}(ds, du) = ds\,\nu_s(du)$. The main quantity of interest is the kernel ν_s which controls the number and the size of the jumps around time s.

In [6] the authors are interested in the detection of abrupt changes in the jump measure of X. Based on high-frequency observations $X_{i\Delta_n}$, $i = 0, \ldots, n$, with $\Delta_n \to 0$ they construct a test for a constant ν against the alternative

$$\nu_t^{(n)} = 1_{\{t<\lfloor n\theta_0\rfloor \Delta_n\}}\nu_1 + 1_{\{t\geq\lfloor n\theta_0\rfloor \Delta_n\}}\nu_2.$$

Here the authors face a similar problem as in the classical situation of changes in the mean of a time series, namely that the "change point" θ_0 can only be defined relative to the length of the covered time horizon $n\Delta_n$ which needs to tend to infinity. In general, this problem cannot be avoided as there are only finitely many large jumps on every compact interval, so consistent estimators for the jump measure have to be constructed over the entire positive half-line.

There are other types of changes in the jump behaviour of a process than just abrupt ones, though. In the sequel, we will deal with gradual (smooth, continuous) changes of ν_s and discuss how and how well they can be detected. A similar problem has recently been addressed in [29] who constructs a test for changes in the activity index. Since this index is determined by the

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