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The distribution of the spine of a Fleming–Viot type process

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Abstract

We show uniqueness of the spine of a Fleming–Viot particle system under minimal assumptions on the driving process. If the driving process is a continuous time Markov process on a finite space, we show that asymptotically, when the number of particles goes to infinity, the distribution of the spine converges to that of the driving process conditioned to stay alive forever, the branching rate for the spine is twice that of a generic particle in the system, and every side branch has the distribution of the unconditioned generic branching tree.

MSC: 60G17

Keywords: Fleming–Viot particle system; Spine

1. Introduction

It is well known that, under suitable assumptions, a branching process can be decomposed into a spine and side branches. Heuristically speaking, the spine has the distribution of the driving process conditioned on non-extinction, the side branches have the distributions of the critical branching process, and the branching rate along the spine is twice the rate along any other trajectory.

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We will prove results for the "Fleming–Viot branching process" introduced in [7] that have the same intuitive content. This process is the extreme case of the Moran model introduced in [23] (see [15, Def. 5.12] for the modern discussion). In the Moran model, individuals branch at a (bounded) intensity while in our model, they do not branch at all when they are in the "main part" of the domain but they branch instantaneously when they hit a "small set".

Our results have to be formulated in a way different from the informal description given above because the distribution of the spine of a Fleming–Viot process with a fixed (finite) number of particles does not have an elegant description (as far as we can tell). The asymptotic or limiting version of the historical version of the model seems to exist, when the number of individuals goes to infinity, but it is precisely the process described by our theorems. Hence, our results have to be asymptotic in nature. We will show that the limit of the spine processes, as the number of particles goes to infinity, has the distribution of the driving process conditioned never to hit the boundary. We will also prove that the rate of branching along the spine converges to twice the rate for a generic particle and the distribution of a side branch converges to the distribution of a branching process with the limiting branching rate.

The doubling of the rate of branching along the spine is a well known phenomenon (see, e.g., [14] and the review of literature in that article) but it seems that no good heuristic explanation for the phenomenon is known. Our arguments suggest such an explanation and we give it in Remark 5.9.

Our main results on the asymptotic spine distribution are limited to Fleming–Viot processes driven by continuous time Markov processes on finite spaces. We conjecture that analogous results hold for all Fleming–Viot processes (perhaps under mild technical assumptions).

The literature on branching processes is huge so we will mention only a few key publications. Most of them contain extensive reference lists. A precursor of our model can be found in a paper by Moran [23]. The book by Jagers [21] is a classical treatise on branching processes and their applications to biology. A modern review of continuous time and space branching can be found in a book by Etheridge [15]. The "Evans' immortal particle picture" was introduced in [16]. The "look-down" process was defined by Donnelly and Kurtz in [12]. Modern approaches to the spine can be found in [14] and [20]. Some of the most profound analysis of the genealogical structure of the Moran and related models appeared in [11,17,18,24]. Another key paper in the area is [22].

Our paper is organized as follows. Section 2 contains basic definitions. It is followed by Section 3 proving existence of the spine under very weak assumptions, thus significantly strengthening a similar result from [19]. Section 4 shows that a historical process, in the spirit of [10,12], can be represented as a Fleming–Viot process and satisfies an appropriate limit theorem. Section 5 contains the main theorems on the distribution of the spine, its branching rate, and its side branches. Section 6 shows by example that the results on the spine distribution must have asymptotic character because they do not necessarily hold for a process with a fixed number of particles.

2. Basic definitions

Our main theorems will be concerned with Fleming–Viot processes driven by Markov processes on finite state spaces. Nevertheless we need to consider Fleming–Viot processes with an abstract underlying state space because our proofs will be based on "dynamical historical processes" which are Fleming–Viot processes driven by Markov processes with values in function spaces.