ARTICLE IN PRESS

Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

stochastic processes and their applications

www.elsevier.com/locate/spa

A stochastic partial differential equation model for the pricing of mortgage-backed securities

F. Ahmad[a], B.M. Hambly[b], S. Ledger[c,*]

[a] *Suleman Dawood School of Business, Lahore University of Management Sciences, Lahore, Pakistan*
[b] *Mathematical Institute, University of Oxford, Radcliffe Observatory Quarter, Oxford OX2 6GG, UK*
[c] *Heilbronn Institute, University of Bristol, Howard House, Bristol BS8 1SN, UK*

Received 10 October 2016; received in revised form 11 September 2017; accepted 23 December 2017
Available online xxxx

Abstract

We develop a dynamic structural model for the wealth of individual mortgagors in a mortgage pool. We model the process of default and prepayment and, by taking a limit as the pool size goes to infinity, derive a stochastic partial differential equation (SPDE) which can be used to describe the evolution of the loss process from the pool. We prove existence and uniqueness of solutions to this SPDE and show how our model is able to capture, in a flexible way, the prices of credit risky tranches of mortgage-backed securities under different market conditions.

Keywords: Stochastic PDE; Particle system; Measure-valued process; Mortgage-backed securities

1. Introduction

The market for mortgage-backed securities (MBS) was one of the fastest growing and most important markets in the US financial industry from its launch in the early 1980s until the financial crisis of 2008. The securitisation of mortgages enabled institutions exposed to mortgage risk to convert these risky, non-rated individual loans into securities that became liquid and most of which had (supposedly) low credit risk. This process accelerated after the dot com bubble

* Corresponding author.
E-mail addresses: ferhana.ahmad@lums.edu.pk (F. Ahmad), hambly@maths.ox.ac.uk (B.M. Hambly), sean.ledger@bristol.ac.uk (S. Ledger).

https://doi.org/10.1016/j.spa.2017.12.002
0304-4149/

of 2000 when the US Federal Reserve lowered interest rates on treasury bills and investment bankers found better investment opportunities in the housing market, which was booming. Through buying thousands of mortgage loans, combining them into an MBS and selling the tranches of the MBS to other investors, many investment banks obtained much better returns than treasury bills would have provided. This process of securitisation was further promoted by the Basel II accord which enabled banks to gain capital relief by using this for credit risk mitigation. The complexities of these products, the use of overly simplistic models and the rapid development of the market in MBS played a major role in the subprime mortgage crisis in 2008. Subsequently the market for agency MBS, those with a government backed guarantee, has rebounded with the market returning to pre-crisis levels, while for MBS from private financial institutions, the market remains very small. In the light of the crisis and the recent regrowth of this market, there remains a need to improve the mathematical models for mortgage-backed securities.

The fundamental underlying structure in an MBS is a pool of mortgages. The particular type of MBS is determined by how the interest and principal repayments of the loans in this pool are repackaged for sale to investors. The 'vanilla' version is the pass-through MBS, in which the interest and principal payments from the pool are passed on to investors with the issuer taking a service charge. A more complicated version is the Collateralized Mortgage Obligation (CMO) in which the pool is tranched, that is the income streams are packaged to enable investors with different risk preferences to invest in the MBS. In the pass-through case the risk of default by a mortgagor in the pool is passed on to all investors, whereas in the CMO case this risk is borne differently by the different tranches, with the lower tranches taking the highest risk. It may also be the case that any early principal repayments are given preferentially to the more senior, lower risk, tranches. Our focus in this paper will be on building a model for a mortgage pool and we will illustrate its performance by pricing CMOs.

When considering an MBS pool, a portfolio of mortgages, it is essential to model not only the defaults of the mortgagors, which leads to a reduction in the income stream from the portfolio, but also their prepayment behaviour. A prepayment occurs when the mortgage is repaid in full before its termination. This event may happen for a number of reasons but the consequence is that, although the principal is repaid, the corresponding income stream making up the MBS is removed. There is no accepted standard way to price an MBS due to the combination of uncertainties in the cash flows due to defaults and prepayments. As in other credit settings there are reduced form and structural approaches and our aim will be to develop a relatively simple flexible dynamic structural approach to the modelling for an MBS.

For an individual mortgage Kau et al. discussed the prepayment and default as the underlying source of uncertainty for the first time in [28,27]. They described the mortgage rate as a solution to a partial differential equation using option pricing ideas with the underlying variables being the interest rate and the house price. Other papers that develop structural models are [12,26]. The second approach to modelling residential mortgages is intensity or hazard rate based. Papers following this approach are [18–20,28,43,50].

In the early modelling of mortgage-backed securities, researchers considered either of the two risks involved with these securities separately. They either modelled the right of prepayment by ruling out the possibility of default or considered default only and overlooked the prepayment right of the borrower. In the setting of agency pass through MBS the government backing enables default risk to be ignored and Dunn and McConnell [13] were the first to use option pricing techniques to model these securities. They considered an agency pass through MBS as equivalent to a single mortgage consisting of a portfolio of a non-callable mortgage loan and an American style option.