Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications 128 (2018) 3897–3939

stochastic processes and their applications

www.elsevier.com/locate/spa

Lifschitz singularity for subordinate Brownian motions in presence of the Poissonian potential on the Sierpiński gasket

Kamil Kaleta[a,b,*], Katarzyna Pietruska-Pałuba[a]

[a] *Institute of Mathematics, University of Warsaw, ul. Banacha 2, 02-097 Warszawa, Poland*

[b] *Faculty of Pure and Applied Mathematics, Wrocław University of Science and Technology, Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland*

Received 17 August 2016; received in revised form 4 January 2018; accepted 8 January 2018
Available online 31 January 2018

Abstract

We establish the Lifschitz-type singularity around the bottom of the spectrum for the integrated density of states for a class of subordinate Brownian motions in presence of the nonnegative Poissonian random potentials, possibly of infinite range, on the Sierpiński gasket. We also study the long-time behaviour for the corresponding averaged Feynman–Kac functionals.

MSC: primary 60J75; 60H25; 60J35; secondary 47D08; 28A80

Keywords: Subordinate Brownian motion; Sierpiński gasket; Random Feynman–Kac semigroup; Schrödinger operator; Random potential; Integrated density of states; Eigenvalues; Reflected process

1. Introduction

The integrated density of states is one of central objects in the physics of large-volume systems, especially systems with in-built randomness. The randomness can come from the interaction with external force field, described by its potential V. This leads to the study of

* Corresponding author at: Faculty of Pure and Applied Mathematics, Wrocław University of Science and Technology, Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland.

E-mail addresses: kamil.kaleta@pwr.edu.pl, kkaleta@mimuw.edu.pl (K. Kaleta), kpp@mimuw.edu.pl (K. Pietruska-Pałuba).

https://doi.org/10.1016/j.spa.2018.01.003
0304-4149/

random Hamiltonians, in particular those of Schrödinger type: given a sufficiently regular, possibly random, potential V one considers the operator

$$H := H_0 + V,$$

where H_0 is the Hamiltonian of the system with no potential interaction. The best analysed situation is that of $H_0 = -\Delta$ (in various state-spaces X). The spectrum of H is typically not discrete. Moreover, spectral properties of such infinite-volume (i.e. defined on the whole space X) Schrödinger operators are usually difficult to handle. The notion of the integrated density of states can come to the rescue: it captures some of the properties of the spectral distribution, while being easier to calculate and easier to work with [4, Chapter VI].

Informally speaking, the integrated density of states arises as follows: one considers operators H restricted to a finite volume $\Omega \subset X$, builds empirical measures l_Ω based on the spectra of these operators normalized by the volume of Ω, i.e.

$$l_\Omega(d\lambda) := \frac{1}{\mathrm{Vol}(\Omega)} \sum_{n=1}^{\infty} \delta_{\lambda_n^\Omega}(d\lambda),$$

and then one takes the limit of l_Ω, in appropriate sense, when $\Omega \nearrow X$. The resulting limit (if it exists) is called the integrated density of states (IDS, for short), and will be denoted by l. Same procedure can be performed for random potentials V^ω — in this case one is interested in the almost-sure limiting behaviour of random measures l_Ω^ω. When the potential V^ω exhibits some ergodicity properties then the limit can be nonrandom.

This paper is concerned with random Schrödinger operators with nonnegative Poissonian potentials. In this case, the existence of the nonrandom IDS is a common feature and for $H_0 = -\Delta$ has been proven e.g. in the Euclidean space [14], hyperbolic space [25], the Sierpiński gasket [18], other nested fractals [23]. In all these situations one has the so-called Lifschitz singularity: the rate of decay of the IDS at the bottom of the spectrum is faster than that of the IDS for the system without random external interaction. Note also that the Lifschitz singularity is closely related to the behaviour of the so-called Wiener sausage when $t \to \infty$ (for the sausage asymptotics in the classical case see [7], on fractals see [18,21]).

While the IDS based on the Laplacian is fairly well understood (see e.g. [4,24]), it is not so for the IDS based on nonlocal operators. In the case of Lévy processes on $\mathbb{R}^d$, the existence and asymptotical properties of IDS with Poissonian potentials have been established in [15,16]. Up to date, there were no results concerning the ‘nonlocal IDS’ on irregular sets, such as fractals. Recently, we have proven the existence of the IDS for subordinate Brownian motions on the Sierpiński gasket perturbed by Poissonian potentials with two-argument profiles W that may have infinite range and local singularities [10]. The Lifschitz tail for stable processes on the Sierpiński gasket evolving among killing Poissonian obstacles was derived in [13].

The present paper is meant as the continuation of [10] in the potential case. Under appropriate assumptions on the potential V (expressed in terms of its profile function W, cf. (2.8) below) and the Laplace exponent ϕ of the subordinator S (assumed to be a complete Bernstein function), we analyse the asymptotical behaviour of the IDS based on the generator of the resulting subordinate Brownian motion evolving in presence of the potential V. Our main results are included in Theorems 4.4 and 3.3. They give the respective lower and upper bounds for $l([0, x])$ as $x \to 0^+$. The rate of decay of $l([0, x])$ as $x \to 0^+$ is of order $\mathrm{e}^{-const \cdot x^{-\sigma}}$, where $\sigma > 0$ is a parameter depending on the behaviour of ϕ at zero and on the rate of decay of the potential profile, $W(x, y)$, as the distance between points x, y tends to infinity. The upper and lower bounds require separate assumptions on the process, and the constant σ may be different in the lower and the upper bound.