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Optical solitons with Biswas-Arshed equation by extended trial function method

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ABSTRACT

This paper studies soliton dynamics for Biswas-Arshed model that is considered when group velocity dispersion is negligibly small and self-phase modulation is totally absent. Two nonlinear forms for the model are studied by the aid of extended trial function scheme. Bright, dark and singular soliton solutions emerge from this scheme.

1. Introduction

Optical solitons form the basic ingredient to transfer information across the globe for trans-continental and trans-oceanic distances. There is a wide variety of model that successfully describe this dynamics of soliton formation [1–13]. The basic principle for the sustainment of solitons in optical fibers, metamaterials, PCF and crystals is the existence of a delicate balance between dispersion and nonlinearity. It may so happen that circumstantial situations lead to low group velocity dispersion (GVD) and absence of nonlinearity. Does this mean that solitons cease to exist under such a situation? Biswas and Arshed recently proposed a very innovative idea to circumvent such a crisis situation. This is reflected in their model that is being referred to as Biswas-Arshed equation [5]. This model was proposed during 2018 with two nonlinear forms and they are Kerr and power law. This paper will address both of these models by extended trial function scheme which will lead to the retrieval of bright, dark and singular soliton solutions. The results are to follow in upcoming sections.

2. Governing model

This section will study Biswas-Arshed equation in two forms.

2.1. Form-I

The model with higher order dispersions and absence of self-phase modulation (SPM) is given by [5]:

$$iq_t + a_1 q_{xx} + a_2 q_{xt} + i(b_1 q_{xxx} + b_2 q_{xxt}) = i[\lambda(|q|^2 q)_x + \mu(|q|^2)_x q + \theta\, |q|^2 q_x]. \tag{1}$$

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In Eq. (1), the first term represents temporal evolution while a_1 and a_2 are the coefficients of GVD and spatio-temporal dispersion (STD). Then, b_1 and b_2 account for third order dispersion (3OD) and third order spatio-temporal dispersion. These dispersion terms will accommodate the low count of GVD. Then with absence of SPM, the nonlinearity effect stems from the coefficients of λ, μ and θ which arise from self-steepening effect and nonlinear dispersions respectively. Thus these compensatory effects of dispersion and nonlinearity provide the necessary balance to sustain soliton propagation.

2.1.1. Mathematical preliminaries

To begin with the integration process, one supposes:

$$q(x,t) = g(s)e^{i\phi(x,t)}, \tag{2}$$

where g denotes the amplitude portion of the wave and

$$s = x - \mathrm{vt}, \tag{3}$$

where v is the soliton speed. The phase component is structured as:

$$\phi(x,t) = -\kappa x + \omega t + \theta_0. \tag{4}$$

where κ is the frequency, ω is the wave number and θ_0 is the phase constant.

Next, plug in (2) into (1). The real part gives:

$$(a_1 - a_2 v + 3b_1\kappa - 2b_2 v\kappa - \omega b_2)g'' - (\omega + a_1\kappa^2 + b_1\kappa^3 - a_2\omega\kappa - b_2\omega\kappa^2)g = (\lambda + \theta)\kappa g^3, \tag{5}$$

where $g' = dg/ds$ and $g'' = d^2g/ds^2$. Then, the imaginary part implies:

$$(b_1 - b_2 v)g''' + (b_2 v\kappa^2 + 2b_2\omega\kappa - 3b_1\kappa^2 - v - 2a_1\kappa + 2a_2 v\kappa + a_2\omega)g' = (3\lambda + 2\mu + \theta)g^2 g'. \tag{6}$$

Upon integrating with respect to s once and choosing the integration constant to be zero leads to:

$$3(b_1 - b_2 v)g'' + 3(b_2 v\kappa^2 + 2b_2\omega\kappa - 3b_1\kappa^2 - v - 2a_1\kappa + 2a_2 v\kappa + a_2\omega)g = (3\lambda + 2\mu + \theta)g^3. \tag{7}$$

As the function g satisfies (5) and (7), the constraint condition that emerges is:

$$\frac{a_1 - a_2 v + 3b_1\kappa - 2b_2 v\kappa - \omega b_2}{3(b_1 - b_2 v)} = -\frac{\omega + a_1\kappa^2 + b_1\kappa^3 - a_2\omega\kappa - b_2\omega\kappa^2}{3(b_2 v\kappa^2 + 2b_2\omega\kappa - 3b_1\kappa^2 - v - 2a_1\kappa + 2a_2 v\kappa + a_2\omega)} = \frac{(\lambda + \theta)\kappa}{3\lambda + 2\mu + \theta}. \tag{8}$$

provided that

$$\mu = \frac{\nu_2(a_2 v - a_1 + b_2\omega) - 6b_1\kappa\lambda - b_2 v\kappa\nu_1}{2(a_1 - a_2 v + 3b_1\kappa - b_2(2v\kappa + \omega))}, \tag{9}$$

and

$$\mu = -\frac{\omega\nu_2 + 2a_2\kappa(\theta\omega + 3v\kappa\nu_3) + \kappa(3v\nu_3(b_2\kappa^2 - 1) + \kappa(\nu_5(b_2\omega - a_1) - 2b_1\kappa\nu_4))}{2(\omega + \kappa(a_1\kappa + b_1\kappa^2 - \omega(a_2 + b_2\kappa)))}, \tag{10}$$

where

$$\nu_1 = \theta - 3\lambda, \quad \nu_2 = \theta + 3\lambda, \quad \nu_3 = \theta + \lambda, \quad \nu_4 = 4\theta + 3\lambda, \quad \nu_5 = 5\theta + 3\lambda. \tag{11}$$

The two expressions, (9) and (10), kick in the condition

$$\frac{\nu_2(a_2 v - a_1 + b_2\omega) - 6b_1\kappa\lambda - b_2 v\kappa\nu_1}{a_1 - a_2 v + 3b_1\kappa - b_2(2v\kappa + \omega)} + \frac{\omega\nu_2 + 2a_2\kappa(\theta\omega + 3v\kappa\nu_3) + \kappa(3v\nu_3(b_2\kappa^2 - 1) + \kappa(\nu_5(b_2\omega - a_1) - 2b_1\kappa\nu_4))}{\omega + \kappa(a_1\kappa + b_1\kappa^2 - \omega(a_2 + b_2\kappa))} = 0. \tag{12}$$

Eq. (5) will be now analyzed by extended trial function scheme (ETFS) [6–9] under the condition (12).

2.1.2. Extended trial function method

To start off, the solution structure to (5) is given by

$$g = \sum_{j=0}^{\varsigma} \gamma_j \Phi^j, \tag{13}$$

where

$$(\Phi')^2 = \Theta(\Phi) = \frac{\Gamma(\Phi)}{\Upsilon(\Phi)} = \frac{\mu_\sigma\Phi^\sigma + \dots + \mu_1\Phi + \mu_0}{\chi_\rho\Phi^\rho + \dots + \chi_1\Phi + \chi_0}. \tag{14}$$

Here $\gamma_0, \dots, \gamma_\varsigma$; $\mu_0, \dots, \mu_\sigma$ and $\chi_0, \dots, \chi_\rho$ are unknown coefficients that will be fixed later such that γ_ς, μ_σ and χ_ρ are non-zero constants. Eq. (14) can be reformulated with an integral form as below: