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Review

New extension of Burr type X distribution properties with application

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ABSTRACT

We introduce a new distribution named Exponentiated Generalized Burr type X (EGBX) distribution with four parameters. This new model can be expressed as a mixture of Burr type X (BX) distribution with different parameters. Several sub models are investigated and also some important structure properties of new model are derived including the quantile function, limit behavior, the *r*th moment, the moment-generating function, Rényi entropy, and order statistics. We estimate the parameters using the maximum likelihood estimation. Finally, simulation study is carried at under varying sample size to assess the performance of this model, the flexibility of EGBX distribution is illustrated by using real data set.

Contents

1. Introduction

Recently, several attempts have been established and studied to define new models that extend the baseline distributions. Burr (1942) introduced twelve distributions using differential equation. Burr type X with one parameter (BX1) and Burr type XII distributions have received much attention in the literatures. Surles and Padgett (2001) proposed a new extension for Burr type X one parameter by adding scale parameter named Burr type X with two parameters or Burr type X (BX) distribution. Many authors have studied widely BX distribution and applied them in different

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areas such as Burr (1942), Merovci et al. (2016),Raqab and Kundu (2006), Shayib and Haghighi (2011) and Merovci et al. (2016) and many others. The cumulative distribution function (cdf) of BX is given by:

$$F(x;\eta,\vartheta) = \left[1 - e^{-(\eta x)^2}\right]^{\vartheta}, \quad \eta > 0, \vartheta > 0, \tag{1}$$

and the probability density function (pdf) of the BX distribution corresponding to the (1) is:

$$f(x;\eta,\vartheta) = 2\,\vartheta\,\eta^2 x e^{-(\eta x)^2}\left[1 - e^{-(\eta x)^2}\right]^{\vartheta-1}, \quad x > 0\eta > 0, \vartheta. \tag{2}$$

The hazard rate functions $h(x)$ of the BX distribution is:

$$h(x;\eta,\vartheta) = 2\,\vartheta\,\eta^2 x e^{-(\eta x)^2}\left[1 - e^{-(\eta x)^2}\right]^{\vartheta-1}\left\{1 - \left[1 - e^{-(\eta x)^2}\right]^{\vartheta}\right\}^{-1},$$

and the r th moment for BX distribution is given as:

$$E(X^r) = \mu^{(r)} = \frac{\vartheta}{\eta^r}\Gamma\left(\frac{r}{2}+1\right)\sum_{k=0}^{\infty}\binom{\vartheta-1}{k}\frac{(-1)^k}{(k+1)^{\frac{r}{2}+1}}.$$

Many families have been established in recent years, Eugene et al. (2002) proposed and study a general class of distributions based on the logic of a beta random variable named Beta-G family distribution and added two shape parameters. Zografos and Balakrishnan (2009) and Ristic and Balakrishnan (2012) used gamma class of family. In Cordeiro et al. (2013) proposed the Exponentiated Generalized (EG) class of distribution with two extra shape parameters. The term of "Exponentiated" and "Generalized" are mean the process of transforming a quantity to some positive real number. The new class of distribution gives more flexibility for the baseline of distribution. The cumulative distribution function of EG is given by:

$$G(x;\alpha,\beta) = \left\{1 - [1 - F(x)]^{\alpha}\right\}^{\beta}, \quad \alpha > 0, \beta > 0, \tag{3}$$

and the probability density function of the EG distribution corresponding to the (3) is:

$$g(x;\alpha,\beta) = \alpha\beta f(x)[1 - F(x)]^{\alpha-1}\left\{1 - [1 - F(x)]^{\alpha}\right\}^{\beta-1}. \tag{4}$$

The first motivation is based on the two extra shape parameters (α,β) of EG family which place a very important role in generating a distribution with heavier tail. The second motivation is the EG family can be applied more effectively on censored incomplete data because its more tractable the beta-G family. Therefore, the new model can analyze continuous univariate and multivariate sets.

In this paper, we propose a new extension of Burr type X distribution with more flexibility than the baseline (2). The new distribution can be applied to different kind of data because the three shape parameters can control the tail of data. The new distribution has more sub-models when compared with baseline distribution and hence it allows us to study more comprehensive structural properties.

Thus, the aims of this work are to explore and study the mathematical properties of the new distribution, which is an extended BX distribution and to prove the new model is more flexible than other models by applying it to real data using a goodness of fit test for real data.

The rest of this paper is arranged as follows: In Section 2, The cumulative function, density function and hazard function of the new distribution are defined. The expansion including the pdf and cdf are provided in Section 3. In Section 4, some mathematical properties of the new model are studied and discussed, such as the quantile function, limit behavior, the r th moment, the moment-generating function, Rényi entropy and order statistics. The parameters of new distribution are estimated using the maximum likelihood method in Section 5. In Section 6, two real datasets are used to illustrate the usefulness of the new model. Finally, concluding remarks are presented in Section 7.

2. Exponentiated Generalized Burr type X

In this section, the four parameter Exponentiated Generalized Burr type X (EGBX) distribution is examined. Several authors have used the EG method to study and explore many distributions. Exponentiated Generalized Inverse Weibull (EGIW) distribution is proposed by using the cdf of Inverse Weibull distribution. In Oguntunde et al. (2015) used the cdf of Weibull distribution to propose the Exponentiated Generalized Weibull (EGW) distribution by applying the Eq. (3). Andrade et al. (2015) used cdf of Gumbel distribution to find a new generalization of Gumbel distribution named Exponentiated Generalized Gumbel (EGGu) distribution.

Let $F(x)$ be the cdf of Burr type X distribution. Inserting (1) in (3) we have the cdf of the new extension of Burr type X distribution named Exponentiated Generalized Burr type X (EGBX) distribution as given in Eq. (5)

$$G(x;\alpha,\beta,\eta,\vartheta) = \left(1 - \left\{1 - \left[1 - e^{-(\eta x)^2}\right]^{\vartheta}\right\}^{\alpha}\right)^{\beta}, \quad \alpha > 0, \beta > 0, \tag{5}$$

where $\alpha,\beta,\vartheta > 0$ are three shape parameters and $\eta > 0$ is scale parameter. The corresponding pdf of (5) for the EGBX distribution is written by:

$$g(x;\alpha,\beta,\eta,\vartheta) = 2\alpha\beta\eta^2\vartheta x e^{-(\eta x)^2}\left[1 - e^{-(\eta x)^2}\right]^{\vartheta-1}\left\{1 - \left[1 - e^{-(\eta x)^2}\right]^{\vartheta}\right\}^{\alpha-1} \times \left(1 - \left\{1 - \left[1 - e^{-(\eta x)^2}\right]^{\vartheta}\right\}^{\alpha}\right)^{\beta-1} \tag{6}$$

$X \sim \text{EGBX}(\alpha,\beta,\eta,\vartheta)$ denotes a random variable with the pdf (6). The hazard function $h(x;\alpha,\beta,\eta,\vartheta)$ of X is given by:

$$h(x;\alpha,\beta,\eta,\vartheta) = 2\alpha\beta\eta^2\vartheta x e^{-(\eta x)^2}\left[1 - e^{-(\eta x)^2}\right]^{\vartheta-1} * \frac{\left\{1 - \left[1 - e^{-(\eta x)^2}\right]^{\vartheta}\right\}^{\alpha-1}\left(1 - \left\{1 - \left[1 - e^{-(\eta x)^2}\right]^{\vartheta}\right\}^{\alpha}\right)^{\beta-1}}{1 - \left(1 - \left\{1 - \left[1 - e^{-(\eta x)^2}\right]^{\vartheta}\right\}^{\alpha}\right)^{\beta}}.$$

The survival function $S(x;\alpha,\beta,\eta,\vartheta)$ of X is given by:

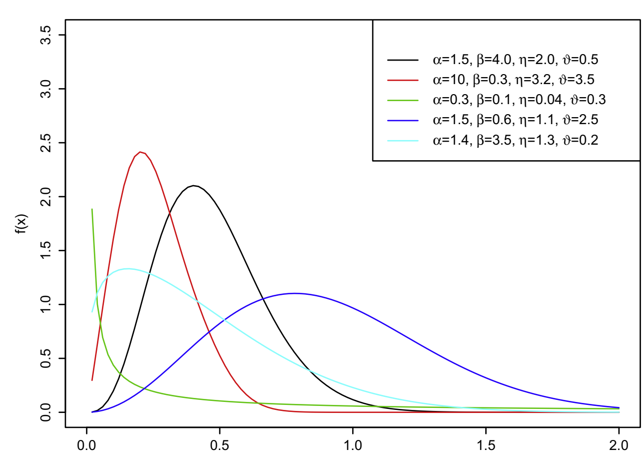

Fig. 1. Plot of the EGBX density function for different values of α,β,η and ϑ.

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