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On s-weakly gw-closed sets in w-spaces

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ABSTRACT

The purpose of this note is to introduce the notion of s-weakly gw-closed set in w-spaces and to study its some basic properties. In particular, the relationships among wg-closed sets, w-semi-closed sets and s-weakly g-closed sets are investigated.

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1. Introduction

In (Siwiec, 1974), the author introduced the notions of weak neighborhoods and weak base in a topological space. We introduced the weak neighborhood systems defined by using the notion of weak neighborhoods in (Min, 2008). The weak neighborhood system induces a weak neighborhood space which is independent of neighborhood spaces (Kent and Min, 2002) and general topological spaces (Császár, 2002). The notions of weak structure and w-space were investigated in (Kim and Min, 2015). In fact, the set of all g-closed subsets (Levine, 1970) in a topological space is a kind of weak structure. We introduced the notion of gw-closed set in (Min and Kim, 2016a) and some its basic properties. In (Min, 2017), we introduced and studied the notion of weakly gw-closed sets for the sake of extending the notion of gw-closed sets in w-spaces. The purpose of this note is to extend the notion of gw-closed sets in w-spaces in a different way than the notion of weakly gw-closed sets. So, we introduce the new notion of s-weakly gw-closed sets in weak spaces, and investigate its properties. In particular, the relationships among weakly wg-closed sets, w-semi-closed sets and s-weakly g-closed sets are investigated.

2. Preliminaries

Let S be a subset of a topological space X. The closure (resp., interior) of S will be denoted by clS (resp., $intS$). A subset S of X is called a *pre-open* (Mashhour et al., 1982) (resp., α-open (Njastad, 1964), *semi-open* (Levine, 1963)) set if $S \subset int(cl(S))$ (resp., $S \subset int(cl(int(S))), S \subset cl(int(S))$). The complement of a pre-open (resp., α-open, *semi-open*) set is called a *pre-closed* (resp., α-closed, *semi-closed*) set. The family of all pre-open (resp., α-open, semi-open) sets in X will be denoted by $PO(X)$ (resp., $\alpha(X), SO(X)$). The δ-interior of a subset A of X is the union of all regular open sets of X contained in A and it is denoted by $\delta - int(A)$ (Velicko, 1968). A subset A is called $\delta - open$ if $A = \delta - int(A)$. The complement of a $\delta - openset$ is called $\delta - closed$. The $\delta - closure$ of a set A in a space (X, τ) is defined by $\{x \in X : A \cap int(cl(B)) \neq , B \in \tau andx \in B\}$ and it is denoted by $\delta - cl(A)$. A subset A of a space (X, δ) is said $a - open$ (Ekici, 2008) if $A \subseteq int(cl(\delta - int(A)))$ and $a - closed$ if $A \subseteq cl(int(\delta - cl(A)))$. And A is said ω^**-open* (Ekici and Jafari, 2010) if for every $x \in V$, there exists an open subset $U \subseteq X$ containing x such that $U - \delta - int(A)$ is countable. The family of all a-open (resp., ω^*-open) sets in X will be denoted by $aO(X)$ (resp., $\omega^* O(X)$).

A subset A of a topological space (X, τ) is said to be:

(a) g-closed (Levine, 1970) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X;
(b) gp-closed (Noiri et al., 1998) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X;
(c) gs-closed (Arya and Nori, 1990) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X;
(d) $g\alpha$-closed (Maki et al., 1994) if $\tau^\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X where $\tau^\alpha = \alpha(X)$;

And the complement of a g-closed (resp., gp-closed, gs-closed, $g\alpha$-closed) set is called a g-open (resp., gp-open, gs-open, $g\alpha$-open) set. The family of all g-open (resp., gp-open sets, gs-open, $g\alpha$-open) sets in X will be denoted by $GO(X)$ (resp., $GPO(X)$, $GSO(X)$, $G\alpha O(X)$).

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Let X be a nonempty set. A subfamily w_X of the power set $P(X)$ is called a *weak structure* (Kim and Min, 2015) on X if it satisfies the following:

(1) $\emptyset \in w_X$ and $X \in w_X$.
(2) For $U_1,\ U_2 \in w_X,\ U_1 \cap U_2 \in w_X$.

Then the pair (X, w_X) is called a *w-space* on X. Then $V \in w_X$ is called a *w-open* set and the complement of a w-open set is a *w-closed* set.

Then the family $\tau,\ \alpha(X),\ GO(X),\ aO(X),\ \omega^*O(X)$ and $g\alpha O(X)$ are all weak structures on X. But $PO(X),\ SO(X),\ GPO(X)$ and $GSO(X)$ are not weak structures on X.

Let (X, w_X) be a w-space. For a subset A of X, the *w-closure* of A and the *w-interior* (Kim and Min, 2015) of A are defined as follows:

(1) $wC(A) = \cap\{F : A \subseteq F,\ X - F \in w_X\}$.
(2) $wI(A) = \cup\{U : U \subseteq A,\ U \in w_X\}$.

Theorem 2.1. [Kim and Min, 2015] *Let (X, w_X) be a w-space and $A \subseteq X$.*

(1) $x \in wI(A)$ if and only if there exists an element $U \in W(x)$ such that $U \subseteq A$.
(2) $x \in wC(A)$ if and only if $A \cap V \neq \emptyset$ for all $V \in W(x)$.
(3) If $A \subseteq B$, then $wI(A) \subseteq wI(B)$; $wC(A) \subseteq wC(B)$.
(4) $wC(X - A) = X - wI(A)$; $wI(X - A) = X - wC(A)$.
(5) If A is w-closed (resp., w-open), then $wC(A) = A$ (resp., $wI(A) = A$).

Let (X, w_X) be a w-space and $A \subseteq X$. Then A is called *a generalized w-closed set* (simply, gw-closed set) (Min and Kim, 2016a) if $wC(A) \subseteq U$, whenever $A \subseteq U$ and U is w-open. If the w_X-structure is a topology, the generalized w-closed set is exactly a generalized closed set in sense of Levine in (Levine, 1970). Obviously, every w-closed set is generalized w-closed, but in general, the converse is not true.

And A is called *a weakly generalized w-closed set* (simply, weakly gw-closed set) (Min, 2017) if $wC(wI(A)) \subseteq U$ whenever $A \subseteq U$ and U is w-open. Obviously, every gw-closed set is weakly gw-closed. In (Min, 2017), we showed that every w-pre-closed set (Min and Kim, 2016b) is weakly gw-closed.

3. Main results

Now, we introduce an extended notion of gw-closed sets in w-spaces as the following:

Definition 3.1. Let (X, w_X) be a w-space and $A \subseteq X$. Then A is said to be *s-weakly generalized w-closed* (simply, s-weakly gw-closed) if $wI(wC(A)) \subseteq U$ whenever $A \subseteq U$ and U is w-open.

Obviously, the next theorem is obtained:

Theorem 3.2. *Every gw-closed set is s-weakly g-closed.*

Remark 3.3. In general, the converse of the above theorem is not true. Furthermore, there is no any relation between s-weakly gw-closed sets and weakly gw-closed sets as shown in the examples below:

Example 3.4. Let $X = \{a, b, c\}$ and $w = \{\emptyset, \{a\}, \{b\}, X\}$ be a weak structure in X. For a w-open set $A = \{b\}$, note that $wI(A) = A$, $wC(A) = \{b, c\}$ and $wI(wC(A)) = wI(\{b, c\}) = A$. So A is s-weakly gw-closed but not gw-closed. And since $wC(wI(A)) = \{b, c\}, A$ is also not weakly gw-closed.

Example 3.5. For $X = \{a, b, c, d\}$, let $w = \{\emptyset, \{d\}, \{a, b\}, \{a, b, c\}, X\}$ be a structure in X. Consider $A = \{a\}$. Then since $wI(A) = \emptyset$, obviously A is weakly gw-closed. For a w-open set $U = \{a, b\}$ with $A \subseteq U, wI(wC(A)) = wI(\{a, b, c\}) = \{a, b, c\} \neg\subseteq U$. So A is not s-weakly gw-closed.

In general, the intersection as well as the union of two s-weakly gw-closed sets is not s-weakly gw-closed as shown in the next examples:

Example 3.6. For $X = \{a, b, c, d\}$, let $w = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, c, d\}, X\}$ be a weak structure in X.

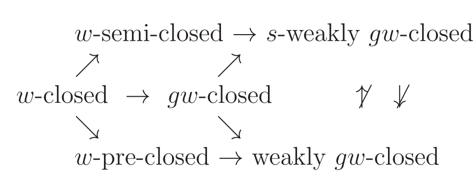
(1) Let us consider $A = \{a\}$ and $B = \{c\}$. Note that $wI(wC(A)) = wI(\{a, d\}) = A, wI(wC(B)) = wI(\{c, d\}) = B$ and $wI(wC(A \cup B)) = wI(\{a, c, d\}) = \{a, c, d\}$. Then we know that A and B are all s-weakly gw-closed sets but the union $A \cup B$ is not s-weakly gw-closed.
(2) Consider two s-weakly gw-closed sets $A = \{a, b, c\}$ and $B = \{a, c, d\}$. Then $A \cap B = \{a, c\}$ is not s-weakly gw-closed in the above (1).

Theorem 3.7. *Let (X, w_X) be a w-space. Then every w-semi-closed set is s-weakly gw-closed.*

Proof. Let A be a w-semi-closed set and U be a w-open set containing A. Since $wI(wC(A)) \subseteq A$, obviously it satisfies $wI(wC(A)) \subseteq U$. It implies that A is s-weakly gw-closed. □

Remark 3.8. In (2) of Example 3.6, the s-weakly gw-closed set $A = \{a, b, c\}$ is not w-semi-closed. So, the converse of the above theorem is not always true.

From the above theorems and examples, the following relations are obtained:



Let X be a nonempty set. Then a family $m(\subseteq P(X))$ of subsets of X is called *a minimal structure* (Maki, 1996) if $\emptyset, X \in m$.

Theorem 3.9. *Let (X, w_X) be a w-space. Then the family of all s-weakly gw-closed sets is a minimal structure in X.*

Lemma 3.10. [Kim and Min, 2015] Let (X, w_τ) be a w-space and $A, B \subseteq X$. Then the following things hold:

(1) $wI(A) \cap wI(B) = wI(A \cap B)$.
(2) $wC(A) \cup wC(B) = wC(A \cup B)$.

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