ARTICLE IN PRESS

Journal of King Saud University – Science xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Journal of King Saud University – Science

journal homepage: www.sciencedirect.com

Original article

One and two spin-1/2 particle systems under the Lorentz transformations

H. Moradpour [a], M. Bahadoran [b], P. Youplao [c,d,*], P. Yupapin [c,d], A. Ghasemi [a]

[a] Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P.O. Box 55134-441, Maragha, Iran
[b] Department of Physics, Shiraz University of Technology, 31371555 Shiraz, Fars, Iran
[c] Department for Management of Science and Technology Development, Ton Duc Thang University, District 7, Ho Chi Minh City, Viet Nam
[d] Faculty of Electrical & Electronics Engineering, Ton Duc Thang University, District 7, Ho Chi Minh City, Viet Nam

ARTICLE INFO

Article history:
Received 1 March 2017
Accepted 24 April 2017
Available online xxxx

Keywords:
Quantum optics
Quantum information
Spin communication
Lorentz transformation
Non-locality

ABSTRACT

Lorentz transformation (LT) was used to link two inertial frames, consisted of moving and lab frames. In addition, the effects of LT on the states of two and one spin-½ particle systems are addressed. Throughout the paper, we only consider two spin operators including Czachor's and the Pauli spin operators. It is shown that the system's state predictions made by Pauli spin operator for one spin-½ particle systems is better than that of made by Czachor's spin operator. Thereinafter, we focused on entangled systems consisted of two spin-½ particles moving away from each other and the treatment of system state under Lorentz transformation was studied .We also use both Pauli and Czachor's operators to build the Bell's operator. Additionally, we address the behavior of Bell's inequality under LT and compare the results made by considering Pauli's operator with that of from Czachor's spin operator. In the last part, some results of considering the Pauli-Lubanski spin operator are also addressed.

1. Introduction

In quantum mechanics, systems may blurt a non-local behavior from themselves (Einstein et al., 1935). Bohm and Aharanov provided a spin version for exhibiting this behavior (Bohm and Aharonov, 1957). In their setup, non-locality leads to entanglement, i.e. the state of the system is not equal to the product of its constituent particles' states (Moradpour et al., 2015). Firstly, Bell tried to get a criterion for distinguishing the local and non-local phenomenon from each other (Bell, 1964). His work leads to a well-known inequality called the Bell inequality which may be violated by non-local states. In fact, there are various models for this inequality (Clauser et al., 1969; Audretsch, 2008; Brunner et al., 2014; Bertlmann, 2014). In the two-particle systems, the Bell operator is defined as

$$B = a \otimes (b + b') + a' \otimes (b - b'). \tag{1}$$

where (a, a′) and (b, b′) are yes or no operators applying on the first and second particles, respectively. For every local state, the Bell operator meets the $\langle B \rangle \leqslant 2$ condition. Some forehand experimental attempts have been done to detect non-locality can be found in (Aspect et al., 1981; Aspect et al., 1982a,b). It is shown that non-locality is not limited to the multi-particle systems and indeed, a one-particle system may also behave non-locally (Dunningham and Vedral, 2007; Cooper and Dunningham, 2008). Non-locality is a source for entropy which has vast implications in current science (Nielsen and Chuang, 2002). It has also been shown that it may be a source of the entropy of horizons in the gravitational and cosmological setups (Das et al., 2008).

Spin is a quantum mechanical property of systems which was exhibited in investigating the relativistic quantum mechanical systems. Pauli derived an operator for describing the spin of particles in the low-velocity limit. By considering the low-velocity limit, Pauli got 2×2 matrixes, called the Pauli matrixes or operator σ_i, and the corresponding spin operator for spin-1/2 particles (Greiner, 1990). Nowadays, it is believed that the predictions from

* Corresponding author at: Department for Management of Science and Technology Development, Ton Duc Thang University, Ho Chi Minh City, District 7, Viet Nam and Faculty of Electrical and Electronics Engineering, Ton Duc Thang University, District 7, Ho Chi Minh City, Viet Nam.

E-mail addresses: h.moradpour@riaam.ac.ir (H. Moradpour), bahadoran@sutch.ac.ir (M. Bahadoran), phichai.youplao@tdt.edu.vn (P. Youplao), preecha.yupapin@tdt.edu.vn (P. Yupapin).

Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.jksus.2017.04.008
1018-3647/© 2017 The Authors. Production and hosting by Elsevier B.V. on behalf of King Saud University.

Please cite this article in press as: Moradpour, H., et al.. Journal of King Saud University – Science (2017), http://dx.doi.org/10.1016/j.jksus.2017.04.008

the Pauli spin operator (S_i) about the spin of systems are in line with the Stern-Gerlach type experiments in the lab frame, a frame in which the particle's velocity is not relativistic (Sakurai and Napolitano, 2014). But, is it the only candidate for the spin operator which leads to the consistent results with a Stern-Gerlach type experiment in the lab frame? Moreover, what is the result of a Stern-Gerlach type experiment, if it is observed by a moving observer which moves with respect to the lab frame with a constant velocity (β)? Indeed, there are various attempts to get a candidate for describing spin and thus the results of applying a Stern-Gerlach type experiment on a system which is in relative motion with respect to observer (Bauke et al., 2014a,b; Caban, 2012; Caban et al., 2013; Czachor, 1997a,b; Terno, 2003), where Czachor followed the Pryce (1948) and Fleming (1965) arguments to get the spin operator as

$$\vec{A}\cdot\vec{S}=(\sqrt{1-\beta_0^2}\vec{A}_{\perp}+\vec{A}_{\parallel})\cdot\vec{S} \tag{2}$$

In addition, the normalized spin operator can be obtained by dividing the above operator into its Eigenvalues and the following normalized operator is achieved for the spin-½ particle which commutes with the Hamiltonian (Czachor, 1997a,b)

$$\hat{A}=\frac{\left(\sqrt{1-\beta_0^2}\vec{A}_{\perp}+\vec{A}_{\parallel}\right)\cdot\vec{\sigma}}{\sqrt{1+\beta_0^2\left[\left(\hat{e}\cdot\vec{A}\right)^2-1\right]}} \tag{3}$$

Based on this result, this operator may be used instead of the Pauli operator along the $\vec{A}$ vector ($\vec{A}\cdot\vec{\sigma}$) whenever, states with zero momentum uncertainty are taken into account. It is easy to check that, independent of $\vec{A}$, the Pauli spin operator along the $\vec{A}$ vector is recovered by substituting $\beta_0=0$. Here, σ and $\hat{e}$ are the Pauli operator and the unit vector along the β_0 direction, respectively (Moradpour et al., 2015). In fact, β_0 represents the particle's velocity, but, since the lab frame is a frame in which the particle's velocity is not relativistic, the Pauli operators are suitable operators to describe the system's spin in the lab frame and thus, we can consider $\beta_0=0$ in the lab frame (Doyeol et al., 2003; Kim and Son, 2005; Moradi, 2008). We should also mention here that, for a moving observer which moves with respect to the lab frame with velocity β, since motion is a relative concept, we have $\beta_0=\beta\neq0$ (Friis et al., 2010; Moradi, 2009; Moradi and Aghaee, 2010; Moradi et al., 2014; Saldanha and Vedral, 2012a,b; Saldanha and Vedral, 2013). Therefore, from now we consider $\beta_0=\beta$ as the boost velocity and $\hat{e}$ as the unit vector directed along the boost direction. It is worthwhile mentioning that the subscripts $\perp$ and $\parallel$ denote the perpendicular and parallel components of the vector $\vec{A}$ to the boost direction, respectively (Moradpour and Montakhab, 2016). This operator also supports the Pauli spin operator either $\vec{A}_{\perp}=0$ or $\vec{A}_{\parallel}=0$ ($\hat{A}=\vec{A}\cdot\vec{\sigma}$). It is worth to note that the uncertainty principle leads to $\Delta\beta\neq0$ and therefore, this principle prevents such possibility in a realistic experiment (Czachor, 1997a,b), where its generalization to the wave-packets can be found. Some of the shortcomings and strengths of Czachor's and the Pauli operators are investigated in (Bauke et al., 2014a,b). Although, just the same as the Pauli operator, Czachor's spin operator should indeed be defined as $\hbar\hat{A}/2$ to coverer the spin-½ particles, we should note that the eigenvalues of Czachor's spin operator are not always equal to $\pm\hbar/2$. Whenever the effects of considering high velocities such as the probability of pair production are ignored, the phenomena interpretations made by quantum mechanics are satisfactory and the lab frame is connected to the moving frame, which moves with a constant velocity with respect to the lab frame, by a LT (Halpern, 1968). Therefore, one may apply LT on the system state in the lab frame to get state seen by the moving observer. By this approach, the spin state of the system is affected by a rotation of the Wigner angle (Wigner and Halpern, 1939). The effects of LT on the single-particle entangled states are investigated by Palge et al. (2011). It is shown that such rotations may also affect the spin entropy of one spin-½ particle as well as the two spin-½ entangled particles systems (Dunningham et al., 2009; Peres et al., 2002; Nishikawa, 2008). There are also various attempts in which authors investigate the behavior of non-locality under LT. Their results can also be used to get some theoretical predictions about the outcome of a Stern-Gerlach type experiment which may lead to getting a more suitable spin operator. The acceleration effects on non-locality are also investigated in León and Martín-Martínez (2009), Mann and Villalba (2009), Smith and Mann (2012), Terashima and Ueda (2004).

Some authors have used the Pauli spin operator to generate the Bell operator and considered bipartite pure entangled state (Terashima and Ueda, 2002, 2003). Thereinafter, they considered a special set of measurement directions which leads to violating Bell's inequality to its maximum violation amount $2\sqrt{2}$ in the lab frame. In addition, they have been considered a moving observer connected to the lab frame by an LT, and applied an LT on the system state in the lab frame to get the corresponding state in the moving frame. They took into account the same set of measurement directions for the moving frame as the lab frame, and investigate the behavior of Bell's inequality in the moving frame. In fact, they use Bell's inequality as a witness for the bi-partite non-locality. Finally, they find that the violation of Bell's inequality in the moving frame is decreased as a function of the boost velocity and the particle energy in the lab frame. It should be noted that if one applies LT on both of the Bell operator and the system state, Bell's inequality is violated to the same value as the lab frame. The generalization of this work to three-particle non-local systems can be found in Moradpour et al. (2015), Moradpour and Montakhab (2016).

In a similar approach, Ahn et al. have been considered the Bell states and used Czachor's operator to construct the Bell operator (Doyeol et al., 2003; Moradpour and Montakhab, 2016). Bearing in mind this fact that Czachor's and the Pauli operators are the same operators in the lab frame ($\beta=0$), authors have considered the special set of spin measurements which violates Bell's inequality to its maximum violation amount in the lab frame. They applied LT on the system state in the lab frame to get the corresponding state in the moving frame. They also assumed that the moving frame uses the same set of spin measurements as the lab frame for evaluating Bell's inequality. Therefore, their setup has some similarity with those of Terashima and Ueda (2002, 2003). There are also some differences between setups investigated in these papers. Their LT differs from each other, and they used the different spin operator to build the Bell operator. Finally, Ahn et al. found out that the expectation value of the Bell operator in the moving frame is decreased as a function of the boost velocity and the energy of particles in the lab frame. It should be noted again that Bell's inequality will be violated in the moving frame to the same value as the lab frame, if one applies LT on both of the Bell operator and the system state (Friis et al., 2010). It means that, the moving observer can obtain the maximum amount of violation for the Bell's inequality provided that, in the moving frame, the LT has been applied to both Bell's states and Bell's operator (Doyeol et al., 2003). More studies on this subject and its generalization to the three-particle non-local systems can be found in Moradi (2008), Moradpour and Montakhab (2016).

In fact, both of the mentioned approaches found out that the expectation value of the Bell operator in the moving frame is decreased by increasing the boost velocity and the energy of parti-