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Two-dimensional analysis of interlaminar stresses in thin anisotropic composites subjected to inertial loads by regularized boundary integral equation

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ABSTRACT

Evaluation of interlaminar stresses in composites plays a crucial role to ensure structural integrity. It is quite often to have composites consisting of very thin anisotropic multilayers. Traditional domain modeling of ultra-thin multilayers usually requires a tremendous amount of refined elements that might cause computation overloading. This article proposes an efficient computational methodology for evaluation of two-dimensional interlaminar stresses in thin anisotropic composites subjected to inertial loads. This analysis is by the regularized boundary integral equation (BIE) that employs only very coarse meshes. In the present work, the directly transformed boundary integral equation is regularized using the scheme of integration by parts and analytical integration. By the proposed approach, modeling of very thin layered composites can be performed simply by very coarse mesh. The obvious advantage of the present method over conventional methods is the much less modeling efforts that are required for analyzing very thin multi-layered composites. For verifications, a few benchmark examples are presented in the end.

1. Introduction

For various purposes, composites are usually made of different phases of anisotropic materials. Multilayered composites are often constructed by bonding together different layers of materials, either isotropic or anisotropic. Due to different properties of adjoin materials, significant stresses shall arise at bonding interfaces under operational environments. As a consequence, there is a crucial concern about the potential de-bonding caused by sever interlaminar stresses and thus, analysis of the interlaminar stress appears to be an important issue. In general, traditional domain modeling of thin multi-layered composites usually requires a tremendous amount of refined elements that often lead to heavy computational burdens. The BEM treatment of three-dimensional (3D) anisotropic structures involves complicated computations of the associated Green's function. In engineering practice, although the 3D modeling appears to be very general for most problems, the 2D modeling of plane stress/strain analysis is still a pretty common practice for simplification in engineering applications. It is apparent that BEM treatment of 3D thin anisotropic structures has been another important challenging task that is worthy of more research work. The present work is to propose an efficient, yet powerful approach for accurately computing interlaminar stresses in ultra-thin layers of two-dimensional composites subjected to inertial loads due to rotations and self-weight.

In engineering practice, thin multi-layered composites have been widely applied for their high strength-to-weight ratio [1]. In the literature, there have been many works proposed to evaluate interfacial stresses of layered composites. Although some analytical solutions exist for very particular problems (e.g. Refs. [2] [3]), recourse to numerical tools, such as the finite difference method (FDM) and the finite element method (FEM), is generally necessary for more general cases with complicated geometry and boundary conditions. There are too many works to mention for a thorough literature review and thus, only a few among them are reviewed here for examples. Lajczok [4] has applied the FDM to the strains and curvatures obtained from MSC/NASTRAN to determine their derivatives, which are incorporated into the classical thin plate theory for calculation of interlaminar shear stresses. Using a 2D FEM scheme, Tolson [5] calculated interlaminar stresses in laminated composites and predicted the progressive failure of the structure. Lo et al. [6] used a higher-order shear deformation theory to find the transverse stress components. The reader may refer to the work by Kant and Swaminathan [7] for a selective review and survey of current development on the estimation of transverse/interlaminar stresses in laminated composites.

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When applied to the stress analysis of ultra-thin composites, the domain solution techniques (FDM and FEM) will confront with the modeling difficulty that a tremendous amount of refined meshes are generally required to yield reliable results. This is because the aspect ratios of discretized meshes are constrained to have orders not less than 1/20 to yield reliable results. As a consequence, the presence of ultra-thin adhesive layers are usually neglected for the FEM modeling of multilayered composites. However, the analysis for adhesive layers can be crucial for accurate assessment of the integrity of composites because failure of composites is often initiated from the fracture of adhesives. Despite the wide recognition of the boundary element method (BEM) as a powerful alternative tool, pertinent BEM researches on the interlaminar stresses of adjoined anisotropic composites have remained comparably scarce indeed. As is well known in the BEM community, an additional domain integral will arise in the boundary integral equation (BIE) for treating inertial loads. As a major drawback, evaluation of this additional domain integral shall require domain discretization that will destroy the BEM's distinctive notion of boundary discretization. For 2D problems, an analytically exact transformation of this domain integral into surface integrals has been proposed by Zhang et al. [8] to recover the BEM as a true boundary solution technique; however, additional line integrals are involved (see Fig. 1) for resolving the problem of discontinuity along the branch cuts of logarithmic complex functions. Obviously, this addition of an extra line integral is very tedious that requires a robust code for correctly determining the intersected regions for all source points on boundary. For this, Shiah and Ye [9] have presented a BIE for treating body-forces in 2D anisotropic materials, where the domain integral is exactly transformed to surface ones without additional line integrals involved. Recently, a new approach was proposed by Wen et al. [10] to transform domain integral to the boundary. Sharing the similar manner as in the DRM, this technique uses radial base functions to approximate the body force term. Also, Gao [11] has proposed the radial integration method for evaluation of domain integrals with boundary discretization. The recently emerging meshless methods have also been discussed for treating some singularity problems (e.g. Ref. [12–14]). Since the present article is only targeted on regularizing the exactly transformed boundary integral equation, no further reviews of other methodologies are provided here.

The present work is targeted on regularizing the newly presented BIE to treat thin layered composites subjected to the inertial loads of self-weight and rotations. In treating thin structures by the BEM, the problem of "nearly singular integrals" will arise when the source point on one side of the structure is very close to the integration element on its opposite side. For correctly computing the integrals, several schemes have been proposed over the years to overcome the numerical difficulty. The most direct approach is to sub-divide the integration elements into several sub-elements as presented in Ref. [15]; however, there will come up another issue regarding the computational efficiency. For that, regularization of the integrals appears to be more efficient such that the conventional numerical integration schemes are capable of yielding reliable results. In this article, a semi-analytical approach is presented for regularizing the integrals in the transformed BIE. Even, this regularization enables the proposed approach to model ultra-thin adhesive layers that are usually neglected in the conventional analysis. In contrast with the domain solution techniques that require proper mesh discretization for thin domains, the BEM approach for such analysis has the merit that ultra-thin layers can be modeled simply by very coarse boundary meshes, yet without any thickness limitation for all layers. Also, this approach may be applied to analyze thin-film coatings, although not in the scope of the present study. In fact, the source of the difficulty of BEM modeling lies in nearly singular integrals when the source point approaches the element under integration for ultra-thin structures. Over the years, various schemes have been proposed to overcome the numerical difficulty of evaluating nearly singular integrals. To name a few as examples, Cruse and Aithal [16] proposed a semi-analytical approach using Taylor series expansions for the kernels. Huang and Cruse [17] presented another approach taking a coordinate transformation to relax nearly singular kernels. Other approaches include Gaussian integration with fine subdivisions, kernel cancellation methods [18], the auxiliary surface of "tent'" method [19], and the line integral method [20–22]. Shiah and Chong [23] have applied a self-regularization technique to study the interior solution of 3D thermoelastic problems. Also, there are far more references regarding this topic and the mentioned works are simply examples. However, most works are presented for isotropic materials and very scarce works on anisotropic structures are found in the open literature. Very recently, Wang and Sun [24] derived a non-singular boundary integral equation (BIE) with the technique of integration by parts to analyze cracked anisotropic bodies. In this article, the nearly singular integrals for conventional elastic analysis without body force are integrated by parts, while those associated with the inertial effects are integrated analytically. In the end, the veracity and the feasibility of the proposed approach are demonstrated by a few illustrative examples.

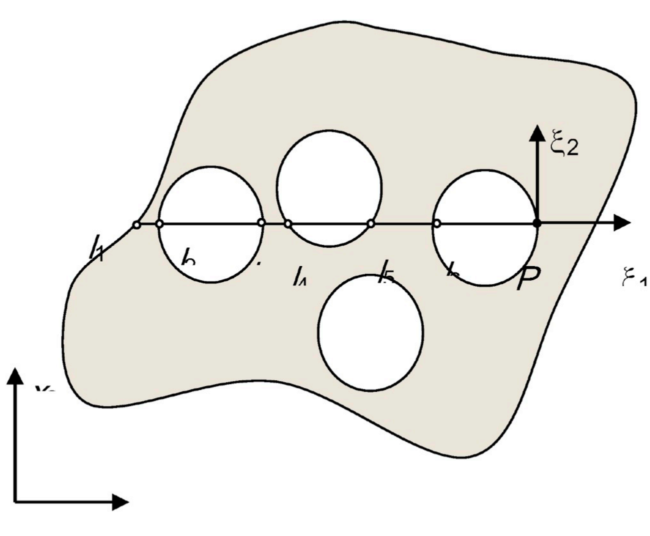

Fig. 1. The branch cut of a source point on the boundary of a multiply connected domain.

2. BIE for 2D anisotropic elasticity

As is well established for 2D anisotropic elasticity, the constitutive relationship between the stress σ_{ij} and the strain ε_{ij} is governed by

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix}=\begin{bmatrix}c_{11}&c_{12}&c_{16}\\ c_{12}&c_{22}&c_{26}\\ c_{16}&c_{26}&c_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{Bmatrix},\ \begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{Bmatrix}=\begin{bmatrix}a_{11}&a_{12}&a_{16}\\ a_{12}&a_{22}&a_{26}\\ a_{16}&a_{26}&a_{66}\end{bmatrix}\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix} \tag{1}$$

where the coefficients c_{mn} and a_{mn} are the elastic stiffness and compliance constants of the material, respectively. These compliances may be given in terms of engineering constants as follows:

$$a_{11}=1/E_1,\quad a_{22}=1/E_2,\quad a_{12}=-\nu_{12}/E_1=-\nu_{21}/E_2,\ a_{16}=\eta_{12,1}/E_1=\eta_{1,12}/G_{12},\quad a_{26}=\eta_{12,2}/E_2=\eta_{2,12}/G_{12},\quad a_{66}=1/G_{12} \tag{2}$$

where E_k is the Young's modulus in the direction of the x_k-axis and G_{12} is the shear modulus on the x_1-x_2 plane; ν_{ij} is the Poisson's ratio, and $\eta_{i,jl}$, $\eta_{ij,l}$ are the coefficients of mutual influence of the first and second kind, respectively. Equation (2) is also applicable to the case of plane strain, provided b_{jk} is substituted for a_{jk} by

$$b_{jk}=a_{jk}-a_{j3}a_{k3}/a_{33},\quad (j,\ k=1,\ 2) \tag{3}$$

where, with the index 3 referring to the x_3-axis, a_{m3} are given by