Electrical Power and Energy Systems 106 (2019) 138–145

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Laplace-based computation of transient profiles along transmission lines including time-varying and non-linear elements

R. Nuricumbo-Guillén[a,*], L. Vergara[b], P. Gómez[c], F.P. Espino-Cortés[a]

[a] Departamento de Ingeniería Eléctrica de SEPI-ESIME Zacatenco, Instituto Politécnico Nacional, Mexico City, Mexico
[b] Centro Nacional de Control de Energía, Mexico City, Mexico
[c] Electrical and Computer Engineering Department, Western Michigan University, Kalamazoo, MI, USA

ARTICLE INFO

Keywords:
Numerical Laplace transform (NLT)
Switching transient
Surge arrester
Transmission line modeling

ABSTRACT

In this paper, a combination of a transmission line model defined in the *q-s* domain (spatial frequency-temporal frequency), the two-dimensional Laplace transform and the principle of superposition is applied to compute transient voltage and current profiles along transmission systems including time-dependent and non-linear elements. The detailed internal information provided by these profiles can be very useful in applications such as insulation coordination design, fault detection and location. The method presented in this paper has significant advantages over the computation of transient profiles using EMTP-type software, given that the latter requires the subdivision of the line, which is time consuming and prone to numerical error accumulation. Additionally, since the line model is defined in the frequency domain, it is possible to include the frequency dependence of the electrical parameters in a very straightforward manner. In order to evidence the accuracy and versatility of the proposed method, several applications examples are presented, and the computed results are compared with those obtained using ATP.

1. Introduction

The simulation of electromagnetic transients is essential for an adequate design of power components and systems [1]. Typical transient analysis of power networks considers terminal (2-port) models of transmission lines, so that voltage and/or current measurements are available at specific nodes or buses. The information provided by such models is considered sufficient for common transient studies. However, since the maximum overvoltages and overcurrents can appear at interior points along transmission lines during a transient event [2], more detailed information may be required for studies of practical significance such as insulation coordination and design, fault detection and location. In such cases, the availability of transient voltages and currents at internal points along the line can be very useful to provide a more comprehensive depiction of wave propagation during phenomena such as direct and indirect lightning strokes, switching operations or faults. A traditional way to obtain measurements at internal points of a line with transient simulation software, such as the Electromagnetic Transients Program (EMTP), consists of dividing the line into several shorter segments connected in series; however, using this approach to model a transmission line is time consuming, can lead to a reduction in accuracy due to numerical error accumulation, and increases the computational resources required. Several methods are available in the literature to obtain internal plots of voltage and/or current, as well as complete profiles along transmission lines given by 3-dimensional plots or contour maps (see for instance [2–7]). Nevertheless, since these methods are defined in the time domain, approximations such as recursive convolution are required to consider the frequency dependence of the line. Alternatively, constant-parameter line models can be used, but this can greatly reduce the accuracy of the computed results.

The use of frequency domain methods can be a good alternative to time domain methods for the computation of transient profiles along transmission lines. A frequency-domain technique for the computation of transient profiles along constant-parameter transmission lines was presented in [8] for applications in electronic systems. More recently, a method based on the successive application of the inverse numerical Laplace transform (NLT) [9,10] to obtain transient voltage and current profiles along transmission systems, including overhead lines, underground cables, transformer windings and electric networks was presented in [11], in which the frequency dependence of the electrical parameters is accounted for by means of the concept of complex penetration depth [12]. This method was further extended to obtain the transient profiles along transmission lines excited by an indirect lightning stroke [13]. It was later demonstrated that the successive

* Corresponding author.
E-mail address: rnuricumbog1600@alumno.ipn.mx (R. Nuricumbo-Guillén).

https://doi.org/10.1016/j.ijepes.2018.09.033
Received 15 March 2018; Received in revised form 3 August 2018; Accepted 20 September 2018

application of the inverse NLT can be replaced by a direct 2-dimensional definition of the inverse NLT (2DNLT) to obtain significant computer savings [14]. In addition, the 2DNLT has been recently applied to produce animations of electromagnetic transients along transmission lines for educational purposes [15,16], as well as to produce contour maps for fault location [17].

However, due to the linearity of the Laplace transform operator, the 2DNLT-based method for computation of transient profiles has been restricted so far to linear time-invariant (LTI) systems. In practice, modern power systems involve many non-linear and time-varying conditions related to the inclusion of switching devices, protective elements, transformers and reactors, non-linear loads, power electronic components, etc. Therefore, the main contribution of this paper is to extend the application of the 2DNLT to allow the computation of transient profiles along multiconductor transmission lines considering time-varying and non-linear conditions by means of the application of the principle of superposition [18].

When compared with transient simulation software such as EMTP, the method presented in this paper has the advantage of straightforward computation of transient profiles without the need to set up a complex model with many line segments and measuring probes. It is also important to note that if the line's characteristics were to change or if there is a need for a different amount of measurement points along the line, a complete modification of the EMTP model would be required, which is impractical and time-consuming. Meanwhile if the proposed method is used, such changes can be taken into consideration by performing a new parameter computation [12] using the updated line characteristics and number of samples, which would require only a few seconds of computer time. An additional advantage of frequency domain methods such as the one presented in this paper is in terms of accuracy, since the inclusion of frequency dependent parameters does not require the numerical approximations needed by time domain methods.

The work presented in [18] has a similar approach than the present paper in the use of the principle of superposition for the consideration of time-dependent and non-linear elements, but it is only able to provide voltage information at the line's terminals. In contrast, the proposed method can compute both voltage and current information at any point along the line.

The potential applications of the work presented in this paper include insulation coordination studies, location of protective devices, fault location algorithms, among others, where transient analysis methods with new capabilities are constantly being developed in order to obtain better results (see for instance [17,22–26]). Several test cases are included to illustrate the capabilities and accuracy of the method, comparing the results with those from the transient simulation program ATP [19].

2. Two-dimensional inverse numerical Laplace transform with hybrid sampling

In previous work where transient profiles have been computed with the inverse NLT [3–5], a conventional (regular) sampling has been used in the algorithm with good results. However, it has been discussed in [1] and [2] that odd sampling can further increase the accuracy of the NLT algorithm. To date, this has not been successfully applied in transient profile computation algorithms due to the fact that odd sampling is not applicable to the spatial domain. This section shows the implementation of a hybrid sampling in the 2DNLT, where a conventional sampling is used for space domain (z), while an odd sampling is used for time domain (t).

The two-dimensional inverse Laplace transform of a real and causal function is defined as

$$f(z,t) \cong \frac{e^{bz+ct}}{4\pi^2} Re\left\{\int_{-R}^{R}\int_{-\Omega}^{\Omega} F(q,s)e^{j(rz+\omega t)}d\omega dr\right\} \tag{1}$$

where $q = b + jr$, $s = c + j\omega$, b, and c are positive real constants, r and ω are the spatial and temporal frequencies, respectively, R and $-R$ are the complex spatial frequency integration limits, and Ω is the temporal frequency upper integration limit. Unlike previous works [13–16] where a regular sampling has been used for both temporal and spatial frequency, the use of a hybrid sampling is proposed in this paper, applying regular sampling in the spatial frequency domain and odd sampling in the temporal frequency domain. Considering an odd sampling of ω ($\Delta\omega$, $3\Delta\omega$, $5\Delta\omega$, $\cdots$) avoids singularities at $\omega = 0$ and, since the spectrum integration step is $2\Delta\omega$, the maximum frequency for odd sampling is twice the maximum frequency of regular sampling, increasing the accuracy of the NLT and improving the attenuation of Gibbs oscillations as mentioned in [10]. The use of odd sampling in the s domain also changes the temporal frequency's lower integration limit to 0. Considering this modification and including a window function $\sigma(r,\omega)$, (1) is modified as follows:

$$f(z,t) \cong \frac{e^{bz+ct}}{2\pi^2} Re\left\{\int_{-R}^{R}\int_{0}^{\Omega} F(b+jr, c+j\omega)e^{j(rz+\omega t)}\sigma(r,\omega) d\omega dr\right\} \tag{2}$$

By applying conventional sampling to the q domain, odd sampling to the s domain (spectrum integration step equal to $2\Delta\omega$), considering an observation time T and a line length L, the following discrete form of (2) is obtained:

$$f_{n_1,n_2} = \frac{e^{(bn_1\Delta z + cn_2\Delta t)}}{2\pi^2} Re\left\{\sum_{m_1=0}^{N_1-1}\sum_{m_2=0}^{N_2-1} F_{\dot{m}_1,\dot{m}_2}\sigma_{0_1,\dot{m}_2} e^{jm_1n_1\Delta r\Delta z} e^{j(2m_2+1)n_2\Delta\omega\Delta t}\Delta r 2\Delta\omega\right\} \tag{3}$$

for $n_i = 0, 1, \cdots, N_i - 1$ and $i = 1, 2$.

where

$$f_{n_1,n_2} = f(n_1\Delta z, n_2\Delta t) \tag{4a}$$

$$F_{\dot{m}_1,\dot{m}_2} = F(b + j\dot{m}_1\Delta r, c + j\dot{m}_2\Delta\omega) \tag{4b}$$

$$\sigma_{\dot{m}_1,\dot{m}_2} = \sigma(b + j\dot{m}_1\Delta r, c + j\dot{m}_2\Delta\omega) \tag{4c}$$

$$\dot{m}_1 = \begin{cases} m_1, & for \quad m_1 = 0, 1, \cdots, \frac{N_1}{2} \\ m_1 - N_1, & for \quad m_1 = \frac{N_1}{2}+1, \cdots, N_1-1 \end{cases} \tag{4d}$$

$$\dot{m}_2 = 2m_2 + 1 \tag{4e}$$

$$b = -\frac{\ln(\varepsilon)}{L} \tag{4f}$$

$$c = -\frac{\ln(\varepsilon)}{T} \tag{4g}$$

ε is a factor with a value between 1×10^{-5} and 1×10^{-3}, N_1 and N_2 are the number of discrete time and space samples, respectively. Also:

$$\Delta r = \frac{2\pi}{L} \tag{5a}$$

$$\Delta\omega = \frac{\pi}{T} \tag{5b}$$

$$\Delta z = \frac{L}{N_1} \tag{5c}$$

$$\Delta t = \frac{T}{N_2} \tag{5d}$$

Using the relations established in (5), it follows that