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On the characterisation of polar fibrous composites when fibres resist bending – Part II: Connection with anisotropic polar linear elasticity☆

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ABSTRACT

This continuation of Part I (Soldatos, 2018) aims to make a connection between the polar linear elasticity for fibre-reinforce materials due to (Spencer and Soldatos, 2007; Soldatos, 2014, 2015) with the anisotropic version and the principal postulates of its counterpart due to (Mindlin and Tiersten, 1963). The outlined analysis, comparison and discussions are purely theoretical, and aim to collect and classify valuable information regarding the nature of continuous as well as weak discontinuity solutions of relevant well-posed boundary value problems. Emphasis is given on the fact that the compared pair of theoretical models has a common theoretical background (Cosserat and Cosserat, 1909) but different kinds of origin. Some new concepts and features, introduced in Part I in association with linear elastic behaviour of materials having embedded fibres resistant in bending, are thus shown relevant to more general linearly elastic, anisotropic, Cosserat-type material behaviour. The different routes followed for the origination of the compared pair of models is known to produce identical results in the case of conventional (non-polar) linear elasticity. The same is here found generally not true in the polar elasticity case, although considerable similarities are also observed. No definite answers are provided regarding the manner in which existing differences might be bridged or, if at all possible, eliminated. These are matters that require further study and thorough investigation.

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1. Introduction

About twenty years after Adkins and Rivlin pioneered the non-linear theory of fibre-reinforced materials (Adkins, 1951; Adkins and Rivlin, 1955), Spencer's (1972) monograph summarised the progress which had been made at the time in the subject. Fig. 1 is extracted from that monograph (Spencer, 1972), and in its initial part (Fig. 1(a, b)) illustrates a cantilevered block of fibre-reinforced material bent in a fully continuous manner. The fibres are noted as *a*-curves and are considered very stiff and strong. Each of the Fig. 1(c) – (i) illustrates next one of many possible analogous deformation patterns that involve different kind of discontinuous fibre slope and/or fibre curvature, although the overall displacement field is still continuous. The example deformation patterns depicted in Fig. 1 underpinned the applicability of the theory of ideal fibre-reinforced materials (Spencer, 1972). Today, these are felt as predictions that, within the elastic deformation regime, justify the term and the class of "weak discontinuity" deformations. The latter are deformation patterns which, due to micro-scale (fibre-thickness) material failure, are described by continuous displacements that possess discontinuous derivatives; see Merodio and Ogden (2002,2003).

Existence of weak discontinuity deformations in non-polar and unconstrained non-linear elasticity did not become formally known before 1975 in the case of material isotropy (Knowles and Sternberg, 1975), and were not studied in connection with fibre-reinforced materials before 1983 (Triantafyllidis and Abeyarante, 1983). Such deformations occur in the form of material instability modes as soon as the influence that large deformation exerts on the elastic constitution of the material forces the equations of elasticity to lose ellipticity. These micro-mechanics failure modes are thus not observable in conventional (non-polar) linear elasticity, where the governing equations are always elliptic.

The same is not necessarily true in the case of polar linear elasticity (e.g., Mindlin and Tiersten, 1963; Spencer and Soldatos, 2007; Soldatos, 2014, 2015) where, still, the magnitude of the deformation does not affect material constitution but, due to the presence of couple-stress, the corresponding governing equations are generally non-elliptic. Weak discontinuity solutions of well-posed boundary value problems in polar linear elasticity may thus co-exist with their fully continuous counterpart(s). The latter are po-

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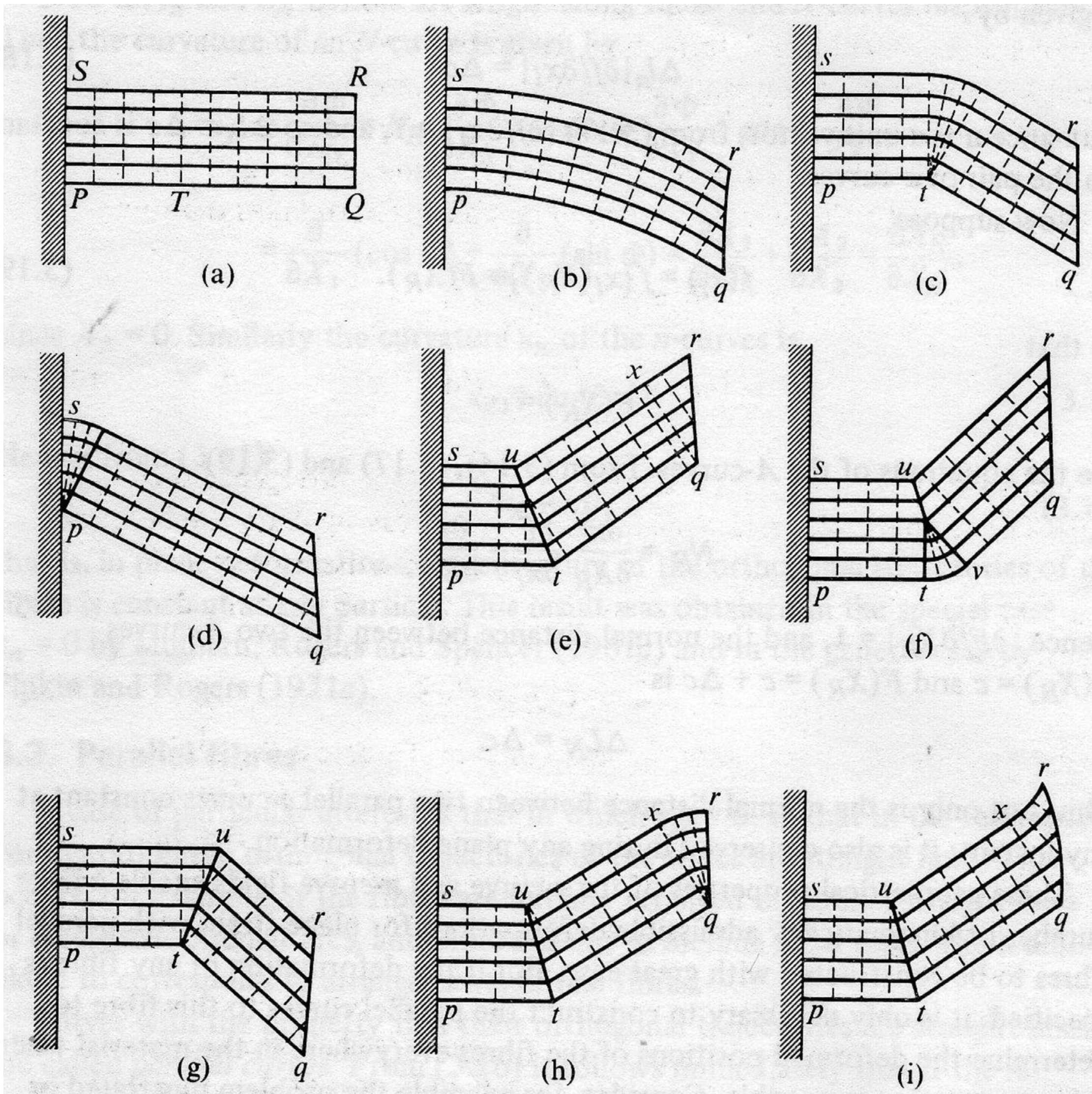

Fig. 1. Scanned image of Fig. 3.2 of (Spencer, 1972) showing: (a) an un-deformed cantilevered rectangular block reinforced by a unidirectional family of straight fibres (the so-called *a*-curves); and (b–i) a number of different deformation patterns, which are kinematically admissible under the theory of ideal fibre-reinforced materials.

tential solutions described by continuous displacements possessing continuous derivatives of all orders, and, for simplicity, will be termed as "continuous solutions" in what follows.

Like the aforementioned monograph (Spencer, 1972), the polar linear elasticity presented by Mindlin and Tiersten (1963) was published before the pioneering work of Knowles and Sternberg (1975) on weak discontinuity elasticity solutions. Mindlin and Tiersten (1963) had thus every reason at the time to claim that a continuous solution to a well-posed mixed boundary value problem formulated in terms of their theory is unique. However, this claim is now disputable, at least because the non-elliptic nature of the relevant governing equations is already exposed and discussed to a certain extent (Gouriotis and Bigoni, 2016).

There exists no evidence suggesting that the anisotropic version of that theory (Mindlin and Tiersten, 1963) was motivated by potential applications on linearly elastic composites with embedded fibres resistant in bending. Moreover, most of the polar linear elasticity analysis detailed in (Mindlin and Tiersten, 1963) deals with the isotropic version of that theory. Hence, a possible rational connection of that theory with applications referring to composites containing fibres resistant in bending would naturally be interesting as well as important in relevant applications (e.g., Asmanoglo and Menzel, 2017).

The present investigation aims to compare the anisotropic version of, and principal postulations stemming from the linear polar elasticity due to Mindlin and Tiersten (1963) with their counterparts presented in Spencer and Soldatos (2007) and Soldatos (2014,2015). The comparison and relevant discussions are currently of purely theoretical nature and significance, and are associated with the search for continuous solutions of well-posed boundary value problems in polar linear elasticity. It is noted in this context that the compared polar elasticity models have a common theoretical background, namely that of the couple-stress theory (Cosserat and Cosserat, 1909) which is summarised in Section 2. However, they have different kind of origin and foundation.

Mindlin and Tiersten's (1963) polar linear elasticity is founded on constitutional considerations stemming from the observation that the internal energy function of the material is quadratic in the strains and the spin-gradients of the deformation. The same constitutive assumptions are thus employed and underpin the generally anisotropic polar linear elasticity formulated in Section 3.1. Nevertheless, several new concepts are introduced in the remain-

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