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Fracture toughness of hierarchical self-similar honeycombs

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ABSTRACT

The influence of self-similar hierarchy on the brittle fracture behavior of a two-dimensional honeycomb is investigated. The honeycomb walls are modeled as Bernoulli-Euler beam elements rigidly connected at the nodal points, and zero, first and second order hierarchies are addressed. The stress state in the vicinity of a semi-infinite crack is determined, and the fracture toughness is calculated in terms of tensile strength of parent material. The growth of hierarchical order leads to increase in the number of nodal degrees of freedom to be taken into account, and the computational cost of the calculations is reduced by employing a novel analysis method based on the discrete Fourier transform.

An increase in hierarchical order enhances the fracture toughness after optimization of geometrical parameters at fixed relative density. This effect becomes more pronounced for honeycombs of lower relative density.

1. Introduction

An efficient way to improve the mechanical performance of solid structural element is to replace the homogeneous material with a material with microstructure. The iteration of this procedure leads to the hierarchical materials possessing remarkable strength, stiffness and toughness. There are a numerous examples of implementation of this design strategy both in nature (e.g., Chen and Pugno (2013); Gibson (2012); Lakes (1993); Gao (2006)) and engineering practice (e.g., Sen and Buehler, 2011; Carpinteri and Pugno, 2008; Lakes, 1993; Taylor et al., 2011; Kooistra et al., 2007; Fan et al., 2008; Mousanezhad et al., 2015a; Mousanezhad et al., 2015b).

Hierarchical materials can be broken down into the two main groups: dense composite materials and cellular ones (Lakes, 1993). Representatives of the first group are the high-rank sequentially laminated composites that have stiffness approaching the theoretical bounds (see Milton (2002) and references there). The bone microstructure represents an example of a high toughness dense hierarchical bio-composite (Currey, 1984). Cellular hierarchical bio-composites (e.g., Gibson, 2012) serve as a source of inspiration for many studies stimulated by recent technology advances that provide an attractive opportunity to manufacture new micro-architectured materials (e.g., Meza et al., 2015; Arabnejad et al., 2016).

There are two main ways to introduce hierarchy in a periodic structure. The first way, which is illustrated in (Fig. 1a), is to consider parent solid material of structural elements as an effective one representing higher order microstructure (e.g., Vigliotti and Pasini, 2013; Zhu and Wang, 2013). Another way (Fig. 1b) is to introduce scaled periodic pattern of the structure at the nodal points where several elements are meet and in this case the original and the scaled patterns may be of the same length scale (e.g., Ajdari et al., 2012; Li et al., 2017). Using this approach, it is possible to obtain fractal-like self-similar honeycombs which are the subject of the present investigation.

The mechanical behavior of self-similar hexagonal honeycombs was studied intensively in several recent works. Their stiffness and strength were investigated in Ajdari et al. (2012); Haghpanah et al. (2013) and Oftader et al. (2014), and linear and non-linear elastic behavior of a spider-web honeycomb was considered in Mousanezhad et al. (2015a). The influence of the self-similar geometry of two-dimensional phononic crystals on their wave propagation behavior was examined in Mousanezhad et al. (2015). However, contrary to non-hierarchical honeycombs (see, for example, Fleck and Qiu, 2007; Kucherov and Ryvkin, 2014), the brittle fracture behavior of hierarchical ones has not been investigated yet and it is the subject of the present paper.

The paper is organized as follows. In the next section, the microstructure of the honeycombs considered is described, and the general scheme of the fracture toughness evaluation is outlined. Section 3 gives the analysis procedure in detail, and it includes also verification examples. Section 4 presents the results of the parametric study. In the final Section 5, concluding remarks are drawn, and the components of the homogeneous $K-$field employed in the analysis are specified in the Appendix.

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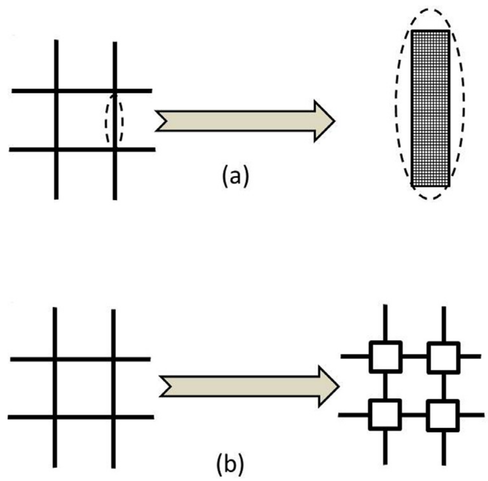

Fig. 1. Two ways of introducing structural hierarchy.

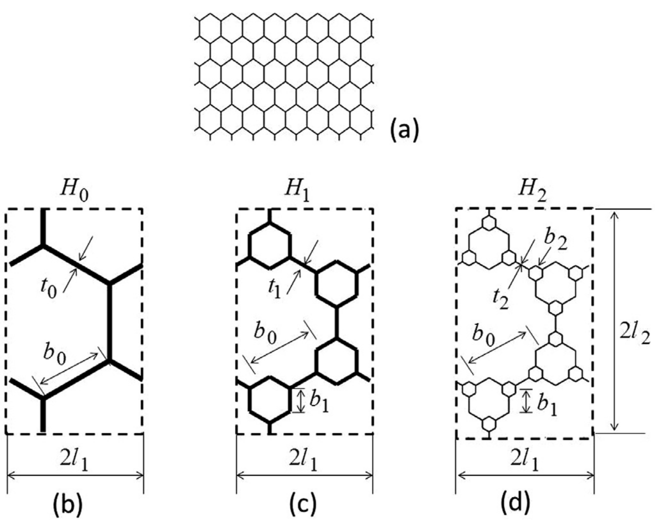

Fig. 2. Self-similar hexagonal hierarchical honeycombs.

2. Problem formulation

2.1. Self-similar hierarchical honeycombs

Consider two-dimensional hexagonal honeycomb composed of beam elements rigidly connected at nodal points (Fig. 2a). The honeycomb has unit out-of-plane thickness, and a case of the plane stress deformation is addressed. The beams satisfy the Bernoulli-Euler assumptions and can undergo bending and axial deformations. The length and thickness of the beams are denoted as b_0 and t_0 respectively, and a periodic pattern of the honeycomb, which is referred as zero-order one H_0, is shown in Fig. 2b. Note that it is possible to use the more primitive unit cell as used in the studies of elastic (e.g., Vigliotti and Pasini, 2012) and fracture properties (Lipperman et al., 2007) of this honeycomb. However, the pattern considered preserves the structure of its boundary with six beam elements approaching it for honeycombs of the higher hierarchical orders, and it is convenient for the development of a uniform analysis procedure.

Higher order self-similar hierarchical honeycombs are obtained from the basic zero-order one by sequential replacement of each node where three elements join with a scaled hexagon. The periodic cells for the first and second order hierarchies H_1 and H_2 are shown in Fig. 2c,d. The non-dimensional parameters for each replacement step i are defined by referring to the initial honeycomb

$$\gamma_i = \frac{b_i}{b_0} \quad \text{and} \quad \eta_i = \frac{t_i}{t_0}\,, \tag{1}$$

where t_i and b_i are the thickness and length of beam elements, respectively. The geometrical constraint of non-overlapping for different hexagons of the nth order in a self-similar hierarchical honeycomb has the form (Haghpanah et al., 2013)

$$\begin{aligned} &\gamma_i \le \gamma_j, \quad i < j\,, \\ &\sum_{i=1}^{n} \gamma_i \le \frac{1}{2}\,, \\ &\sum_{i=j+1}^{n} \gamma_i \le \gamma_j\,, \quad 0 < j < n\,. \end{aligned} \tag{2}$$

Similar to the original H_0 honeycomb, all these microstructures are isotropic on a macroscale thanks to the elastic symmetry, and the Young modulus and Poisson ratio of homogenized material are denoted as E_* and ν_*, respectively. Another essential honeycomb material characteristic is its relative density defined as area fraction of the solid parent material. In the present work, we carry out calculations for honeycombs with zero, first and second order hierarchies. Their relative densities are given as

$$\rho_0 = \frac{2}{\sqrt{3}}\frac{t_0}{b_0} \tag{3}$$

$$\rho_1 = (4\eta_1\gamma_1 - 2\gamma_1 + 1)\frac{2}{\sqrt{3}}\frac{t_0}{b_0} \tag{4}$$

$$\rho_2 = 2[(12\eta_2 - 4\eta_1 - 2)\gamma_2 + 2\gamma_1(2\eta_1 - 1) + 1]\frac{2}{\sqrt{3}}\frac{t_0}{b_0}\,. \tag{5}$$

2.2. The fracture toughness evaluation

The subject of the present investigation is the fracture toughness of the self-similar hierarchical honeycombs. Assuming that brittle fracture and crack advance occur when the maximum local tensile stress in some beam in the crack-tip vicinity attains the tensile strength of the parent material σ_{fs}, it is possible to derive the fracture toughness of a honeycomb K_C theoretically by a numerical experiment (see Schmidt and Fleck, 2001; Fleck and Qiu, 2007). At the outer boundary of the honeycomb, the conditions applied correspond to the K-field for a traction-less semi-infinite crack in the homogeneous material with the effective properties E_*, ν_*. After evaluation of the stress state of the honeycomb, the maximal tensile stress in the crack tip vicinity $\sigma_{\max}$ is determined by comparison for all beams the expression

$$\sigma_{max} = \frac{Mt}{2I} + \frac{N_{ax}}{t}\,. \tag{6}$$

Here, M is an absolute value of the bending moment at a beam extremity, t is the beam thickness, I is moment of inertia of beam's cross-section and N_{ax} is the axial force in the beam element. Then, in view of the problem linearity, one obtains

$$K_C = K\frac{\sigma_{fs}}{\sigma_{max}}, \tag{7}$$

where K is the amplitude of the homogeneous $K-$ field solution, which is presented in the Appendix.

The analysis scheme is illustrated in Fig. 3. Note that the rectangular analysis domain Ω_f must include a sufficiently large number of repetitive cells to provide true values of stresses in the zone of interest at the crack-tip vicinity. Consequently, the condition

$$n_f = \frac{L_f}{l} \gg 1\,, \tag{8}$$