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Coupling effects of dual-phase-lag heat conduction and property difference on thermal shock fracture of coating/substrate structures

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ABSTRACT

This paper firstly studies the heat conduction in a coating/substrate structure subjected to a sudden cooling on the coating surface based on the dual-phase-lag heat conduction model. Then the thermally induced surface cracking problem is solved and the thermal stress intensity factor at the crack tip is evaluated. It is found that the coupling effects of the thermal properties (such as coefficients of thermal conductivity and diffusivity, and phase lags of heat flux and temperature gradient) on the thermoelastic behavior can be predicted by two introduced factors. Besides, the numerical examples reveal that the thermal stress intensity factor decreases with the ratio of substrate elasticity modulus to coating elasticity modulus. Especially if the ratio is greater than 1, namely the substrate is harder than the coating, the peak value of transient thermal stress intensity factor may decrease with the crack length.

1. Introduction

Nowadays, our devices are encountering more and more terrible thermal environments such as satellite in orbit, thermal barrier coatings used in gas-turbine engines (Padture et al., 2002) and reusable launch vehicles (Zoby et al., 2004). Thermal barrier materials and structures are indispensable to the devices work under ultra low (or high) temperature and high heat fluxes (Padture et al., 2002; Zoby et al., 2004; Li et al., 2010). The thermomechanical analysis of materials and structures under severe thermal loadings is essential to the design and application of thermal barrier system (Cheng et al., 2013; 2014a; 2014b).

The thermomechanical studies based on the classic Fourier heat conduction model is abundant in literatures (Erdogan and Wu, 1996; Yu and Qin, 1996; Qin and Mai, 1999; Feng and Wu, 2001; Ma et al., 2011; Feng et al., 2012; Li et al., 2016). The classic Fourier law gives sufficient accuracy in many engineering applications but some severe thermal loadings (Al-Nimr, 1997; Naji et al., 2007). Under the framework of classic Fourier heat transfer, the responses of the heat flux and the temperature gradient are synchronous, which means an infinite heat propagation speed. However, the heat wave (second sound) was discovered in He II (Landau, 1941). Since then the wavelike behavior of heat conduction is observed in more and more heat processes including high heat flux (Maurer et al., 1973), short pulse laser heating (Qiu and Tien, 1993), low temperature (Cimmelli and Frischmuth, 1996), and materials with a nonhomogeneous innerstructure (Kaminski, 1990) and so on.

To describe the wavelike behavior of heat conduction, a modified Fourier law was independently formulated by Cattaneo(1958) and Vernottee (1958), which is so called C-V model. The heat conduction law is

$$\boldsymbol{q}(\boldsymbol{x}, t+\tau_q) = -k_c \nabla T(\boldsymbol{x}, t) \quad (1)$$

where T is the temperature, $\boldsymbol{q}$ is the thermal flux vector, $\boldsymbol{x}$ is the position vector, t is the time, k_c is the thermal conductivity coefficient, τ_q is the thermal relaxation time, and ∇ is the differential operator. The key parameter τ_q is related to the collision frequency of the molecules within the energy carrier. This model has been wildly applied in many fundamental thermal shock fracture analysis of single body (Chen and Hu, 2012; Wang and Li, 2013a; Fu et al., 2014; Guo and Wang, 2015), and structure (Chen and Hu, 2014) and piezoelectric materials (Wang and Li, 2013b; Zhang et al., 2013).

However, Tzou (1995a) pointed out that the concept of macroscopic average implied in the C-V model may lose its physical support in small-scale and high-rate heating. To provide a more accurate approach for the heat conduction in small-scale and the high-rate heating, Tzou (1995a)established the dual-phase-lag (DPL) model based on the hyperbolic two-step model

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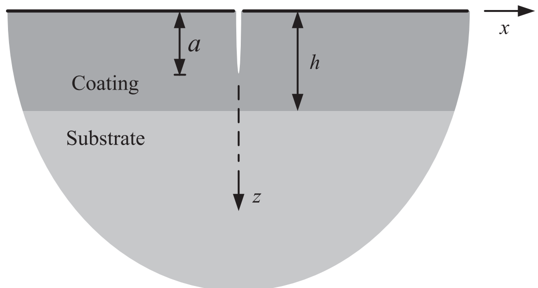

Fig. 1. The geometry of the surface cracked coating/substrate system.

(Anisimov et al., 1974). The heat conduction law is

$$\boldsymbol{q}(t+\tau_q) = -k_c \nabla T(t+\tau_T) \tag{2}$$

where τ_q and τ_T are the phase lags of the thermal flux and the temperature gradient respectively. It is gratifying to see that the DPL model is well supported by the experiment (Tzou, 1995b). Mathematically, the DPL model can be reduced to the C-V model or classic Fourier model in some special cases, thus covering a wide scale of space and time for physical observations (Tzou, 1995a). Since the presentation of DPL model, many researchers have focused on the associated heat conduction problems, but the thermal shock fracture studies based on DPL model is scarce so far.

For most materials especially ceramic, the tensile strength is less than the compressive strength. Thus a crack may initiate at the surface of a medium under cooling shock (Becher et al., 1980; Bahr et al., 1986). Wang et al. (2015) analyzed the thermal shock fracture strength of a surface crack in a half-space based on the DPL model. Furthermore, the coating/substrate structure should be studied because of its wildly application in engineering. This paper considered a coating/substrate structure with a surface crack undergoing a thermal shock on the surface based on the DPL model. It will extend the work accomplished by Rizk and Erdogan (1989) based on the classic Fourier model. The temperature and the associated thermal stress in the un-cracked structure are conducted based on the DPL model first in Section 2. Then the fracture problem of the surface crack under the thermal shock is demonstrated in Section 3. The singular integral technique is used to obtain the thermal stress intensity factor at the crack tip. For illustration purposes, some numerical examples are discussed in Section 4. Finally, the conclusions of this paper are drawn in Section. 5.

2. The thermoelasticity field of un-cracked medium

We consider a surface cracked coating bonded to an infinite substrate. The previous work (Zhong et al., 2009) shows that the study of imperfect interface problem is significant to comprehensive understanding of composites. However, this paper tries to focus on the coupling effects of dual-phase-lag heat conduction and property difference of components on thermoelastic responds of coating/substrate structures. Therefore, for convenience, we consider a ideally bonded coating/substrate structure. The geometry of the coating/substrate structure is depicted in Fig. 1. The length of the crack is a and the thickness of the coating is h. Based on the framework of linear thermoelasticity, the cracking problem of the medium under thermal shock can be simulated by the cracking of the medium under crack surface traction. The thermoelasticity field of the un-cracked coating/substrate system subjected to a thermal shock will be studied in this Section.

The coating and substrate are initially at a constant temperature zero. The surface $x=0$ is suddenly cooled to temperature $-T_0$ at $t=0$ and holds at $-T_0$ for $t>0$. This is a problem of one-dimensional temperature rise in the medium along z direction. The temperature is denoted by the function $T(z, t)$. The interface between coating and substrate is ideal that the temperature and heat flux are continuous at $z=h$ (Ho et al., 2003). Thus the initial and boundary conditions for the temperature field are: $T(z, 0)=0$; $T(0, t)=-T_0 H(t)$, where $H(t)$ is the Heaviside function; $T(h^+, t)=T(h^-, t)$; $q(h^-, t)=q(h^+, t)$; and $T(\infty, t)=0$.

Mathematically, the DPL heat conduction law in Eq. (2) is approached by its Taylor expression:

$$\left(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right)q_z(z,t) = -k_c\left(1+\tau_T\frac{\partial}{\partial t}\right)\frac{\partial}{\partial z}T(z,t) \tag{3}$$

The governing equation of the temperature based on the DPL model is obtained as:

$$\left(\frac{1}{2}\tau_q^2\frac{\partial^3}{\partial t^3}+\tau_q\frac{\partial^2}{\partial t^2}+\frac{\partial}{\partial t}\right)T(z,t) = k_d\left(1+\tau_T\frac{\partial}{\partial t}\right)\frac{\partial^2}{\partial z^2}T(z,t) \tag{4}$$

where k_d is the coefficient of thermal diffusivity, and the inner heat source is ignored. For a systematic research, we introduce dimensionless material parameters $\bar{k}_c = k_c^{(2)}/k_c^{(1)}$, $\bar{k}_d = k_d^{(2)}/k_d^{(1)}$, $\bar{\tau}_q = {\tau_q}^{(2)}/{\tau_q}^{(1)}$, $\upsilon_1 = {\tau_T}^{(1)}/{\tau_q}^{(1)}$, $\upsilon_2 = {\tau_T}^{(2)}/{\tau_q}^{(2)}$, and dimensionless geometry size $\bar{a} = a/l$ and $\bar{h} = h/l$, and dimensionless coordinates $\bar{z} = z/l$ and $\bar{t} = t/\tau_q^{(1)}$. The superscript $^{(1)}$ and $^{(2)}$ indicate the coating and substrate respectively. The characteristic length l is defined as $l = \sqrt{k_d^{(1)}\tau_q^{(1)}}$. Using the dimensionless system, the governing Eq. (4) can be rewritten as:

$$\begin{cases} \left(\frac{1}{2}\frac{\partial^3}{\partial \bar{t}^3}+\frac{\partial^2}{\partial \bar{t}^2}+\frac{\partial}{\partial \bar{t}}\right)T(\bar{z},\bar{t}) = \left(1+\upsilon_1\frac{\partial}{\partial \bar{t}}\right)\frac{\partial^2}{\partial \bar{z}^2}T(\bar{z},\bar{t}), & 0\le\bar{z}\le\bar{h} \\ \left(\frac{1}{2}\bar{\tau}_q^2\frac{\partial^3}{\partial \bar{t}^3}+\bar{\tau}_q\frac{\partial^2}{\partial \bar{t}^2}+\frac{\partial}{\partial \bar{t}}\right)T(\bar{z},\bar{t}) = \bar{k}_d\left(1+\upsilon_2\bar{\tau}_q\frac{\partial}{\partial \bar{t}}\right)\frac{\partial^2}{\partial \bar{z}^2}T(\bar{z},\bar{t}), & \bar{z}\ge\bar{h} \end{cases} \tag{5}$$

Applying Laplace transform with respect to variable $\bar{t}$, the temperature field in Laplace transform domain is deduced from Eqs. (5) and given as:

$$\begin{cases} T^*(\bar{z},p) = A(p)e^{-\lambda_0\bar{z}}+B(p)e^{\lambda_0\bar{z}}, & 0\le\bar{z}\le\bar{h}, \\ T^*(\bar{z},p) = C(p)e^{-\mu_0\bar{z}}+D(p)e^{\mu_0\bar{z}}, & \bar{z}\ge\bar{h}, \end{cases} \tag{6}$$

where the variable with a superscript * indicates its Laplace transformation; the characteristic values are $\lambda_0 = (0.5p^3+p^2+p)^{1/2}(1+\upsilon_1 p)^{-1/2}$ and $\mu_0 = (0.5\bar{\tau}_q^2p^3+\bar{\tau}_q p^2+p)^{1/2}(\bar{k}_d(1+\upsilon_2\bar{\tau}_q p))^{-1/2}$; the functions A, B, C and D are unknowns to be determined by boundary conditions. In Laplace transform domain, the boundary conditions of temperature are rewritten as:

$$\begin{cases} T^*(0,p) = -T_0/p; \\ T^*(h^+,p) = T^*(h^-,p); \\ \frac{1+\upsilon_1 p}{0.5p^2+p+1}\frac{k_c^{(1)}}{l}\frac{\partial}{\partial\bar{x}}T^*(h^-,p) = \frac{1+\upsilon_2\bar{\tau}_q p}{0.5\bar{\tau}_q^2p^2+\bar{\tau}_q p+1}\frac{k_c^{(2)}}{l}\frac{\partial}{\partial\bar{x}}T^*(h^+,p); \\ T^*(\infty,p) = 0. \end{cases} \tag{7}$$

Substituting the temperature expression into boundary conditions above, the functions A, B, C and D are obtained as:

$$\begin{cases} A = -(1+\psi)\Lambda T_0 e^{\lambda_0\bar{h}}, \\ B = -(1-\psi)\Lambda T_0 e^{-\lambda_0\bar{h}}, \\ C = -2\Lambda T_0 e^{-\mu_0\bar{h}}, \\ D = 0, \end{cases} \tag{8}$$

where $\psi = (\bar{k}_c/\bar{k}_d)(\lambda_0/\mu_0)$ and $\Lambda = p^{-1}((e^{\lambda_0\bar{h}}+e^{-\lambda_0\bar{h}})+\psi(e^{\lambda_0\bar{h}}-e^{-\lambda_0\bar{h}}))^{-1}$. If the coating/substrate structure is homogenous half-space medium, the characteristic values satisfy $\lambda_0=\mu_0$ and the factor ψ will be equal to 1, and the expression of the temperature coincides with the previous work (Wang et al., 2015).