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ON EIGENVALUE GENERIC PROPERTIES OF THE LAPLACE-NEUMANN OPERATOR

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Abstract. We establish the existence of analytic curves of eigenvalues for the Laplace-Neumann operator through an analytic variation of the metric of a compact Riemannian manifold M with boundary by means of a new approach rather than Kato's method for unbounded operators. We obtain an expression for the derivative of the curve of eigenvalues, which is used as a device to prove that the eigenvalues of the Laplace-Neumann operator are generically simple in the space $\mathcal{M}^k$ of all C^k Riemannian metrics on M. This implies the existence of a residual set of metrics in $\mathcal{M}^k$, which make the spectrum of the Laplace-Neumann operator simple. We also give a precise information about the complementary of this residual set as well as about the structure of the set of the deformation of a Riemannian metric which preserves double eigenvalues.

1. Introduction

In her seminal work Uhlenbeck [12] proved groundbreaking results on generic properties for eigenvalues and eigenfunctions of the Laplace-Beltrami operator Δ_g on a closed n-dimensional Riemannian manifold (M^n, g). From a qualitative point of view, one of the most beautiful results in [12] is the celebrated Theorem 8 asserting that the set of all C^k Riemannian metrics g for which Δ_g has simple spectrum is residual in the separable Banach space $\mathcal{M}^k$ of all C^k Riemannian metrics on M^n, for any $2 \leq k < \infty$ equipped with the C^k topology. Over the last four decades, similar results were obtained in various directions. We refer to [2, 4, 8] and the references therein for background on this subject.

In line with this theme, Micheletti and Pistoia [9, Theorem 4.1] have proposed a sufficient condition for the set of the deformations of a Riemannian metric g on M^n, which preserve the multiplicity $m \geq 2$ of a fixed eigenvalue $\lambda(g)$ associated with g, to be locally a submanifold of codimension $\frac{1}{2}m(m+1)-1$ inside the Banach space $\mathcal{S}^{2,k}$ of all C^k symmetric covariant 2–tensors on M. They proved that such a condition is easily fulfilled when $n = 2$ and $m = 2$, see [9, Theorem 4.3]. Explicit examples were given which are in accordance with their results. Shortly after, Teytel defined a notion of *meager codimension* in an infinite-dimensional separable Banach space (see [11, Section 2]) that can be used to give a precise information about the set of metrics which the Laplacian has at least one eigenvalue with multiplicity greater than one. The crucial step in approach of Teytel has been to impose a condition,

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