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Persistence of delayed cooperative models: Impulsive control method[☆]

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ABSTRACT

In this paper, the problem of impulsive control for persistence of *N*-species cooperative models with time-varying delays are studied. A method on impulsive control is introduced to delayed cooperative models and some sufficient conditions for the persistence of the addressed models are derived, which are easy to check in real problems. The results show that proper impulsive control strategy may contribute to the persistence of cooperative populations and maintain the balance of an ecosystem. Conversely, the undesired impulsive control such as impulsive harvesting too frequently or impulsive harvesting too drastically may destroy the persistence of populations and leads to the extinction of some species. In addition, some discussions and comparisons with the recent works in the literature are given. Finally, the proposed method is applied to two numerical examples to show the effectiveness and advantages of our results.

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1. Introduction

It is well known that one of the important problems in mathematical biology is to find some conditions which ensure all species in a multispecies community can be persistent [1–3]. It is necessary to keep persistence of populations to maintain the balance of an ecosystem in the real world. During the past two decades, many research work has been done for the persistence of various biological models, see [4–8] and the references cited therein. Cooperation is one of the important interactions among species, which is commonly seen in social animals and in human society [9]. The corresponding mathematical modelling which is called cooperative model usually has a common property that each state variable has a nonnegative influence on the other state variables [10,11]. Many interesting results on dynamics, especially on persistence problem, of various cooperative models have been reported in the past years, see [12–22]. In particular, May [14] proposed

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a class of cooperative models to describe a pair of mutualist as follows:

$$\begin{cases} \dot{u}(t) = r_1 u(t)\left[1 - \dfrac{u(t)}{a_1 + b_1 v(t)} - c_1 u(t)\right], \\ \dot{v}(t) = r_1 v(t)\left[1 - \dfrac{v(t)}{a_2 + b_2 u(t)} - c_2 v(t)\right], \end{cases}$$

where $r_i, a_i, b_i, c_i, i = 1, 2,$ are some positive constants; u and v denote the densities of two species at time t, respectively. Some results on persistence of the models were initially presented from mathematical point of view when u and v are positive variables. And then, various generalized forms of the above models which reflect more realistic dynamic in nature the are being proposed and investigated, for instance, [15,16,19] for the cases of nonautonomous systems and [22] for the discrete cases. In particular, as we know, time delays occur naturally in just about every interaction of the real world which are natural components of the dynamic processes of biology, ecology, physiology, economics, epidemiology and mechanics. For cooperative models, it is more reasonable and realistic to introduce the time delay into the models, since the population of a species in cooperative community depends on not only itself current and past history, but also the current and past history of another species due to the intraspecific cooperative relationship. There have been many studies in literatures that investigate the population dynamics of cooperative models with time delays [20,21].

Systems with impulsive effects describing many evolution processes are characterized by the fact that they are subjected to instantaneous perturbations and experience abrupt changes at certain moments of time which cannot be considered continuously, and there have been significant studies of such systems, see [23–25,30,32,33] and the references therein. Especially, impulsive control as a discontinuous control method has been applied to many practical problems such as orbital transfer of satellite, drug administration, stabilization and synchronization in chaotic secure communication systems, ecological systems and population models see [26–29]. Its extraordinary superiority lie in that, impulses control may change the dynamics of systems, even reverse the dynamics via the perturbations only in some discrete instants, thus control cost and the amount of transmitted information can be reduced greatly. In many cases, moreover, even only impulsive control can be used for control purpose. For instance, a central bank can not change its interest rate everyday in order to regulate the money supply in a financial market; A deep-space spacecraft can not leave its engine on continuously if it has only limited fuel supply [26]; To make the rocket transfer to a higher energy orbit, increments in velocity are given impulsively when the rocket reaches the position of peri-apse and apo-apse [31].

As we know, in the real world, many species in ecosystem are often disturbed by some internal or external factors such as birthrate and deathrate in itself, natural enemies or human activity, that exhibit the impulsive effects, which may lead to the decrease or extinct of some species, see [34–37]. One of the practical problem in ecology is that whether or not an ecosystem can withstand those disturbances and keep persistence over a long period of time. In this sense, the proper impulsive control strategies may contribute to the persistence of populations and maintain the balance of an ecosystem. Conversely, the improper impulsive effects may lead to ecological unbalance and extinction of some species. In recent years, many interesting results on impulsive control of persistence of various biological models have been reported, see [38–44]. For instance, the authors [38] proposed Holling II functional response predator-prey system by periodic impulsive immigration of natural enemies and derived some conditions for extinction of pest and permanence of the system caused by the impulsive control; In [39], a class of impulsive control strategies were given to ensure the permanence and stability of an Ivlev-type predator-prey system based on Floquet theory and comparison principle; In [41], impulsive control for Lotka-Volterra competitive models with time delays were studied by comparison principle and the Lyapunov method. Although there are so many works on impulsive control of biological models, one may note that there is little work on impulsive control of cooperative models with/witout time delays. In fact, extending those impulsive control methods such as those in [38–44] to cooperative systems is quite hard, the reason being that the existing theories of impulsive differential equations is of little help for cooperative systems since each state variable in such systems has a nonnegative influence on the other state variables. Recently, Stamova [45] considered a class of N-species cooperative models with time delays and impulsive control as follows:

$$\begin{cases} \dot{x}_i(t) = x_i(t)\left[r_i(t) - \dfrac{x_i(t-\tau_{ii}(t))}{a_i(t) + \sum_{j=1, j\neq i}^{N} b_j(t) x_j(t-\tau_{ij}(t))} - c_i(t)x_i(t)\right], \\ x_i(t_k) = x_i(t_k^-) + I_{ik}(x_i(t_k^-)), \qquad k \in \mathbb{Z}_+. \end{cases}$$

Some results for persistence and uniform asymptotic stability of the models were derived via Lyapunov Razumikhin method. It shows that the proper impulsive effects such as stocking and harvesting can control the cooperative system's population dynamics. Unfortunately, we have to point out that one of these contributions [45] contains a result which is not correct.

In this paper, we shall present some results on impulsive control of cooperative models with time-varying delays. More precisely, a method on impulsive control is introduced in the context of delayed cooperative models. The rest of this paper is organized as follows: In Section 2, we shall introduce some preliminary knowledge and problem formulation. In Section 3, some new sufficient conditions for persistence of the addressed models are presented. It is shown that proper impulsive control strategies may contribute to the persistence of cooperative populations and maintain the balance of an ecosystem. Also, some comparison with recent works are given in this Section. Two examples and their simulations are illustrated to show the effectiveness and advantages of the result in Section 4. Finally, we shall make concluding remarks in Section 5.

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