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Riemann problem and elementary wave interactions in dusty gas

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A B S T R A C T

The present paper concerns with the study of the Riemann problem for a quasi-linear hyperbolic system of partial differential equations governing the one dimensional isentropic dusty gas flow. The shock and rarefaction waves and their properties for the problem are investigated. We also examine how some of the properties of shock and rarefaction waves in a dusty gas flow differ from isentropic ideal gas flow. The solution of Riemann problem of dusty gas flow for different initial data is discussed. Under certain conditions, the uniqueness and existence of the solution of the Riemann problem has been analyzed. Finally, all possible interactions of elementary waves are discussed.

1. Introduction

Dusty gas is a mixture of gas and small solid particles. The solid particles have a small volume fraction but its density may be large relative to the gas, such types of flow have broad range of application from industrial processes to geophysical flows. There are many processes in which mass fraction of solid particles is very small in comparison to gas such as volcanic jets, solid particle motion in rocket exhaust etc. [1–6]. So the study of Riemann problem for such type of flow plays a major role in the field of mathematics as well as in engineering and physics.

The governing equations describing a planar isentropic dusty gas flow are given by [7]

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0, \tag{1a}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{\partial p}{\partial x} = 0, \tag{1b}$$

where $\rho \geq 0$ is the density of dusty gas, u is the flow velocity, p is the pressure and $t > 0$, $x \in \mathbb{R}$. The specific heat of dusty gas at constant pressure is given by $c_{pd} = k_p c_{sp} + (1 - k_p)c_p$, where c_p and c_{sp} stand for specific heat of gas and specific heat of solid particle respectively and $k_p = m_{sp}/m_{gd}$ is the mass fraction of solid particle in total mass of dusty gas, where m_{sp} is mass of solid particles and m_{gd} is mass of dusty gas. The ratio of specific heats for dusty gas is given by [1,24]

$$\Gamma = \frac{c_{pd}}{c_{vd}} = \frac{\gamma + \delta\beta}{1 + \delta\beta},$$

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where $\delta = k_p/(1-k_p)$, $\beta = c_{sp}/c_p$, $\gamma = c_p/c_v$, c_v specific heat of gas at constant volume. For isentropic dusty gas flow, equation of state is given by [22], $p = k(\rho/(1-Z))^{\Gamma}$, where k is a positive constant and the quantity Z denotes the volume fraction of solid particle in total volume of dusty gas. The relation between the entities Z and k_p is given as $k_p = Z\rho_{sp}/\rho$, where ρ_{sp} stands for species density of solid particles in dusty gas. Since mass fraction of solid particle must be constant in the equilibrium flow therefore z/ρ = constant (say θ).

A detailed analytical and numerical investigation of Riemann problem in different material media has been carried out in [7–16]. Also the problem of interaction of elementary waves for unsteady one dimensional Euler equations was studied by Smoller [9,23] and Chang and Hsiao [17]. Liu [18] discussed the interaction of elementary waves for non linear degenerate wave equations. The Riemann problem for magnetogasdynamics and their wave interactions was studied by Raja Sekhar and Sharma [19], Liu and Sun [20,21].

In present paper, we consider Riemann problem for (1) with initial data given by

$$U(x,0) = \begin{cases} U_l = (\rho_l, u_l) & x < x_0, \\ U_r = (\rho_r, u_r) & x > x_0, \end{cases} \tag{2}$$

where (ρ_l,u_l) and(ρ_r,u_r) are constant states.

In the next sections we will discuss the effect of dust particles on the shock and rarefaction wave and its properties by using the method of characteristics. The existence and uniqueness of the solution under certain conditions will be obtained constructively. The numerical solution of Riemann problem of dusty gas flow for different initial data will be determined. We also discuss all possible interactions of elementary waves.

2. Shock and rarefaction waves

Eq. (1a and b) can be written in conservative form as follows:

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) = 0, \tag{3a}$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial}{\partial x}(p + \rho u^2) = 0. \tag{3b}$$

Eq. (3) may be written in the matrix form as

$$U_t + AU_x = 0, \tag{4}$$

where $U = [{\rho \atop \rho u}]$ and the A is a 2×2 matrix may be written as

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{(\rho u)^2}{\rho^2} + p'(\rho) & \frac{2(\rho u)}{\rho} \end{bmatrix},$$

where prime denotes the differentiation with respect to ρ. The eigenvalues of A are

$$\lambda_1 = u - \left(p'(\rho)\right)^{1/2}, \lambda_2 = u + \left(p'(\rho)\right)^{1/2},$$

where $p'(\rho) = (\Gamma p/(\rho(1-z)))$. The right eigenvectors of A corresponding to eigenvalues λ_1 and λ_2 may be written as

$$r_1 = \begin{bmatrix} 1 \\ \lambda_1 \end{bmatrix}, r_2 = \begin{bmatrix} 1 \\ \lambda_2 \end{bmatrix}.$$

Let $\nabla\lambda_i$ denotes the gradient of eigenvalues λ_i, for $i = 1$, 2. Then

$$\nabla\lambda_i \cdot r_i = \nabla\left(\frac{(\rho u)}{\rho} + (-1)^i\sqrt{p'(\rho)}\right)\left[\frac{1}{\frac{(\rho u)}{\rho} + (-1)^i\sqrt{p'(\rho)}}\right] = (-1)^i\left(\frac{p''(\rho)}{2\sqrt{p'(\rho)}} + \frac{\sqrt{p'(\rho)}}{\rho}\right),$$

which is nonzero quantity therefore both the characteristic fields are genuinely nonlinear. Hence the wave associated with r_1 or r_2 characteristic field is either a shock or a rarefaction wave.

2.1. Shock wave

Let ρ_l, $u_l = u(\rho_l)$, $p_l = p(\rho_l)$ and ρ, $u = u(\rho)$, $p = p(\rho)$ denote respectively the left and right hand states of either a shock or a rarefaction wave. Then the Rankine–Hugoniot (RH) jump conditions for the system (3) are

$$s(u - u_l) = (\rho u - \rho_l u_l), \tag{5}$$

$$s(\rho u - \rho_l u_l) = \left(p + \rho u^2 - p_l - \rho_l {u_l}^2\right), \tag{6}$$