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A flexible terminal approach to stochastic stability and stabilization of continuous-time semi-Markovian jump systems with time-varying delay

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ABSTRACT

This paper addresses the stochastic stability and stabilization problems for a class of semi-Markovian jump systems (SMJSs) with time-varying delay, where the time-varying delay $\tau(t)$ is assumed to satisfy $\tau_1 \le \tau(t) \le \tau_2$. Based on the flexible terminal approach, the time-varying delay $\tau(t)$ is first transformed such that $\tau_1(t) \le \tau(t) \le \tau_2(t)$. By utilizing a novel semi-Markovian Lyapunov Krasoviskii functional (SMLKF) and an improved reciprocally convex inequality (RCI), sufficient conditions are established to guarantee a feasible solution. Two illustrated examples are shown the effectiveness of the main results.

1. Introduction

To date, Markovian jump systems (MJSs) have received much attention due to their merits in describing complex systems with random abrupt variations. MJSs have wide applications in many fields, such as chemical processing, converter applications, aircraft control, robotics, etc. [1–9]. Over the past decades, the research of MJSs has found the framework of stability analysis [10–14], H_∞ control, and filtering [15–18]. It is noted that in [19–24], the transition probabilities (TPs) are time-invariant. However, this assumption is not realistic in many real applications because the TPs in applications are not fixed when the environment suddenly varies. To overcome this shortcoming, nonhomogenous MJSs are introduced in [25,26], where the TPs are allowed to be time-varying. In the reported literature [27,28], the sojourn-time (ST) in nonhomogeneous MJSs obey the exponential probability distribution, which plays a limited role with jump time. To deal with this, SMJSs are proposed, where the TPs are time-varying. Note that SMJSs are more general than MJSs in real situations, because the ST obeys the Weibull distribution instead of an exponential distribution. Thereby, SMJNNs can be employed to model dynamic complex systems, which cover the conventional MJSs as special cases. Results for the stochastic stability analysis and control synthesis of SMJSs are available in [29–35].

Time delay is commonly encountered in various dynamic systems, for instance, networked control systems [36–41], neural networks [42,43], and chaotic systems [44–48]. It is a source of poor performance or instability of dynamic systems. To overcome the shortcomings of the time delay, widespread techniques have been employed, and numerous investigations

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on delay-dependent stability conditions of MJSs have been reported in the literature [49,50]. Note that the stability analysis for MJSs subject to time delay are divided into two types: delay-dependent stability and delay-independent stability. When a time interval is utilized in delay-dependent stability conditions, this may achieve less conservative criteria than with delay-independent stability conditions. Recently, to achieve less conservative delay-dependent stability conditions, efficient augmented LKFs and free-weighting matrix techniques have been used. It can be seen from the existing literature that MJSs with time delays have attracted considerable research [51–54]. However, to our knowledge, little effort has been devoted to the stabilization problem of SMJSs with time-varying delays. The aforementioned work of SMJSs can be improved by reducing conservatism, which is theoretically challenging and open in the research community. This motivates the current study.

The purpose of this study is to propose improved stability and stabilization criteria for SMJSs with time-varying delays for realistic circumstances. By using a flexible terminal approach and a novel SMLKF, improved stochastic stabilization conditions are developed. Finally, two examples are illustrated to depict the effectiveness and less conservatism of the proposed criteria.

Notation: In this work, all matrices are assumed to have proper dimensions. $\mathcal{R}^n$ represents the n dimensional Euclidean space, the symbol * is used as an ellipsis for terms for symmetry. $\mathrm{sym}(Z) = Z + Z^\intercal$. In $(\Omega, \mathcal{F}, \mathcal{P})$, Ω, $\mathcal{F}$, and $\mathcal{P}$, respectively, stand for the sample space, subsets of sample space, and the probability measure. $\mathcal{E}\{\cdot\}$ stands for the expectation operator.

2. Preliminaries

Let us consider the following SMJSs, defined on a fixed probability space:

$$\begin{cases} \dot{x}(t) = A_{r_t}x(t) + A_{\tau r_t}x(t-\tau(t)) + B_{r_t}u(t), \\ x(t) = \phi(t),\ t \in [-\tau_M, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^{n_x}$ is the state vector, $u(t) \in \mathcal{R}^{n_u}$ is the input control, and A_{r_t}, $A_{\tau r_t}$, and B_{r_t} are constant matrices with proper dimensions.

The continuous-time semi-Markov chain r_t takes values of $\mathcal{N}_1 = \{1, 2, \dots, N\}$, subject to the transition probability matrix (TPM) as follows:

$$\Pr(r_{t+h} = q \mid r_t = p) = \begin{cases} \pi_{pq}(h)h + o(h), & p \neq q, \\ 1 + \pi_{pp}(h)h + o(h), & p = q, \end{cases} \tag{2}$$

where $\pi_{pq}(h)$ denotes the transition rate (TR) from mode p at time t to mode q at time $t+h$, and $\pi_{pp}(h) = -\sum_{q\in\mathcal{N}, q\neq p}\pi_{pq}(h)$. $o(h)$ is little-o notation defined by $\lim_{h\to 0} o(h)/h = 0$. $\tau(t)$ is a time-varying functional satisfying

$$0 \le \tau_m \le \tau(t) \le \tau_M, \quad \mu_m \le \dot{\tau}(t) \le \mu_M, \tag{3}$$

where τ_m, τ_M, μ_m, and μ_M are constants.

Remark 1. Inspired by the work [3,55], in this study, to shorten the delay interval artificially, a flexible terminal approach is adopted, namely, $\tau_1(t) = \gamma\tau(t) + (1-\gamma)\tau_m$, $\tau_2(t) = \gamma\tau(t) + (1-\gamma)\tau_M$, where $\gamma \in [0, 1]$, which has been proved to be an efficient way in reducing conservatism of criteria.

For the stability analysis of system (1), the following definition and lemmas are needed.

Definition 2.1. [32] For $\forall\ \phi(t) \in [-\tau_M, 0]$ and $r_0 \in \mathcal{N}$, the SMJS (1) is called stochastically stable, if the following condition is satisfied:

$$\lim_{t\to\infty} \mathbb{E}\left\{\int_0^t \| x(s) \|^2 \, ds \mid (\phi, r_0)\right\} \le \infty.$$

Lemma 2.1. *[39]: For any symmetric matrix $\mathscr{R} > 0$, scalars $\varsigma_1 < \varsigma_2$, with vector ϖ, the following inequality holds:*

$$(\varsigma_2 - \varsigma_1)\int_{\varsigma_1}^{\varsigma_2} \dot{\varpi}^\intercal(s)\mathscr{R}\dot{\varpi}(s)ds \ge \chi_1^\intercal\mathscr{R}\chi_1 + 3\chi_2^\intercal\mathscr{R}\chi_2 + 5\chi_3^\intercal\mathscr{R}\chi_3,$$

where $\chi_1 = \varpi(\varsigma_2) - \varpi(\varsigma_1)$, $\chi_2 = \varpi(\varsigma_2) + \varpi(\varsigma_1) - \frac{2}{\varsigma_2-\varsigma_1}\int_{\varsigma_1}^{\varsigma_2}\varpi(s)ds$, *and* $\chi_3 = \varpi(\varsigma_2) - \varpi(\varsigma_1) - \frac{6}{\varsigma_2-\varsigma_1}\int_{\varsigma_1}^{\varsigma_2}\varpi(s)ds - \frac{12}{(\varsigma_2-\varsigma_1)^2}\int_{\varsigma_1}^{\varsigma_2}\int_s^{\varsigma_2}\varpi(\theta)d\theta ds$.

Lemma 2.2. *[55]Let ϕ and φ be real column vectors with dimensions of n_1 and n_2, respectively. For given real symmetric positive definite matrices $\mathscr{Q}_1$ and $\mathscr{Q}_2$, if $\begin{bmatrix} \mathscr{Q}_1 & \mathscr{R} \\ * & \mathscr{Q}_2 \end{bmatrix} \ge 0$, then the following inequality holds for any scalar $\varrho > 0$ and $\mathscr{R}$:*

$$-2\phi^\intercal\mathscr{R}\varphi \le \varrho\phi^\intercal\mathscr{Q}_1\phi + \varrho^{-1}\varphi^\intercal\mathscr{Q}_2\varphi.$$

Lemma 2.3. *[3]For any matrices $\mathscr{X}_1$ and $\mathscr{X}_2$, symmetric positive definite matrix $\mathscr{Y}$, two functions $\varphi_1(t)$ and $\varphi_2(t)$ satisfying $0 \le \varphi_1 \le \varphi_1(t) \le \varphi_2(t) \le \varphi_2$, and vector function ψ such that $\int_{\varphi_1(t)}^{\varphi_2(t)}\int_\theta^{\varphi_2(t)}\psi(s)dsd\theta = \vartheta_1^\intercal\zeta_1(t)$ and $\int_{\varphi_1(t)}^{\varphi_2(t)}\int_{\varphi_1(t)}^{\theta}\psi(s)dsd\theta =$*